Fluid Mechanics

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Books

- White, F.M., “Fluid Mechanics”, Mc-Graw Hill, 2001.
- **Reference Books**

1. Munson, B.R., “ Fundamental of Fluid Mechanics”, John Wiley, 2002.
2. Cengal Y., Fluid Mechanics”, Mc-Graw Hill, 2001.

What is a Fluid?

- Substances with no strength
- Deform when forces are applied
- Include water and gases

Solid:

Deforms a fixed amount or breaks completely when a stress is applied on it.

Fluid:

Deforms continuously as long as any shear stress is applied.

What is Mechanics?

The study of motion and the forces which cause (or prevent) the motion.

Three types:

- **Kinematics (kinetics):** The description of motion: displacement, velocity and acceleration.
- **Statics:** The study of forces acting on the particles or bodies at rest.
- **Dynamics:** The study of forces acting on the particles and bodies in motion.

Type of Stresses?

Stress = Force /Area

- **Shear stress/Tangential stress:**

  The force acting parallel to the surface per unit area of the surface.

- **Normal stress:**

  A force acting perpendicular to the surface per unit area of the surface.

How Do We Study Fluid Mechanics?

Basic laws of physics:

- Conservation of mass
- Conservation of momentum – Newton’s second law of motion
- Conservation of energy: First law of thermodynamics
- Second law of thermodynamics

+ Equation of state

Fluid properties e.g., density as a function of pressure and temperature.

+ Constitutive laws

Relationship between the stresses and the deformation of the material.

Density and Specific Gravity

- The density ρ of an object is its mass per unit volume:

$$\rho = \frac{m}{V},$$

The SI unit for density is kg/m^3. Density is also sometimes given in g/cm^3; to convert g/cm^3 to kg/m^3, multiply by 1000.

Water at 4°C has a density of $1\ g/cm^3 = 1000\ kg/m^3$.

- The specific gravity of a substance is the ratio of its density to that of water.

Viscosity

It is defined as the internal resistance offered by one layer of fluid to the adjacent layer.

In case of liquids main reason of the viscosity is molecular bonding or cohesion.

In case of gases main reason of viscosity is molecular collision.

- Variation of viscosity with temperature:

In case of liquids, due to increase in temperature the viscosity will decrease due to breaking of cohesive bonds

In case of gases, the viscosity will increase with temperature because of molecular collision increases

Newton's law of viscosity:

This law states that "shear stress is directly proportional to the rate of shear strain".

$$\tau \text{ ά } du/dy$$

$$\tau = \mu du/dy$$

where μ= Dynamic Viscosity having

Unit: SI: $N\text{-}S/m^2$ or Pa-s

CGS: Poise= $\text{dyne-Sec}/cm^2$

1Poise= 0.1 Pa-sec

1/100 poise is called Centipoise.

Note: All those fluids are known as Newtonian Fluids for which viscosity is constant with respect to the rate of deformation.

Kinematic Viscosity (ν)

- It is defined as the ratio of dynamic viscosity to density.

$$\nu = \mu/\rho$$

Units: SI: m^2/s

CGS: Stoke= cm^2/s

1 Stoke= $10^{-4}\ m^2/s$

Note: Dynamic viscosity shows resistance to motion between two adjacent layers where as kinematic viscosity shows resistance to molecular momentum transfer (molecular collision)

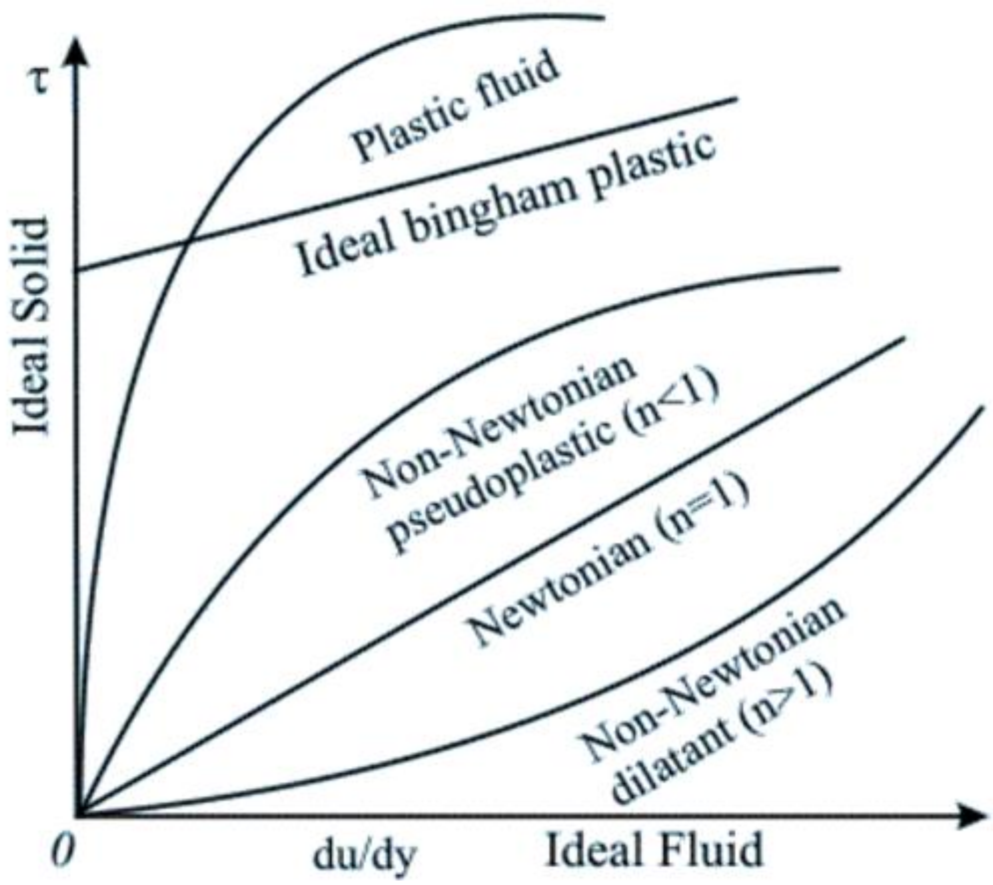
Types of Fluid

- Common fluids, e.g., water, air, mercury obey Newton's law of viscosity and are known as **Newtonian fluid**.
- Other classes of fluids, e.g., paints, polymer solution, blood do not obey the typical linear relationship of stress and strain. They are known as **Non-Newtonian fluids.**

Non-Newtonian Fluids

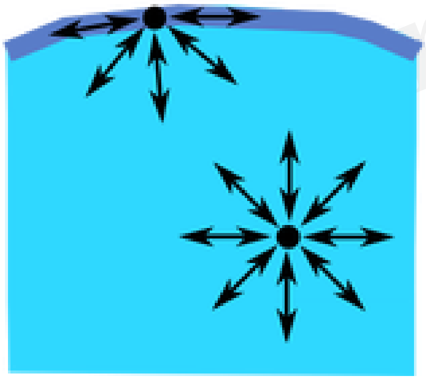
Non-Newtonian Fluid $(\tau \neq \mu \frac{du}{dy})$		
Purely Viscous Fluids		**Visco-elastic Fluids**
Time - Independent	**Time - Dependent**	**Visco- elastic Fluids**
1. Pseudo plastic Fluids $\tau = \mu\left(\frac{du}{dy}\right)^n ; n < 1$ Example: Blood, milk **2. Dilatant Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n ; n > 1$ Example: Butter **3. Bingham or Ideal Plastic Fluid** $\tau = \tau_o + \mu\left(\frac{du}{dy}\right)^n$ Example: Water suspensions of clay and flyash	**1.Thixotropic Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$ *f(t)is decreasing* Example: Printer ink; crude oil **2. Rheopectic Fluids** $\tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$ *f(t)is increasing* Example: Rare liquid solid suspension	$\tau = \mu\frac{du}{dy} + \alpha E$ Example: Liquid-solid combinations in pipe flow.

Shear Stress and Rate of Deformation Relationship for different fluids



Surface Tension

- The surface tension of water provides the necessary wall tension for the formation of bubbles with water. The tendency to minimize that wall tension pulls the bubbles into spherical shapes



Surface Tension

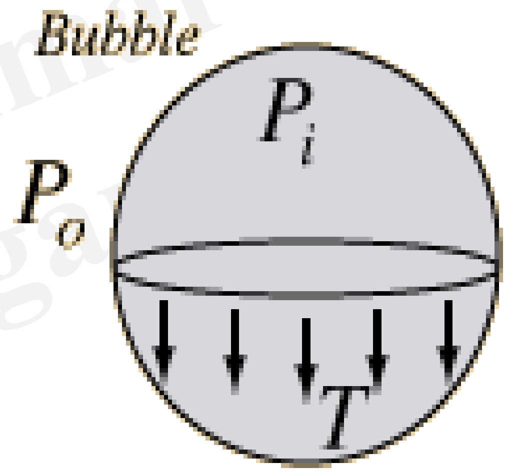
- The pressure difference between the inside and outside of a bubble depends upon the surface tension and the radius of the bubble.

- The relationship can be obtained by visualizing the bubble as two hemispheres and noting that the internal pressure which tends to push the hemispheres apart is counteracted by the surface tension acting around the circumference of the circle.

Surface Tension

$$P_i - P_o = \frac{4T}{r}$$ for a bubble

$$P_i - P_o = \frac{2T}{r}$$ for a droplet which has only one surface.



Surface Tension

- The net upward force on the top hemisphere of the bubble is just the pressure difference times the area of the equatorial circle:

$$F_{upward} = (P_i - P_o)\pi r^2$$

Surface Tension

- The surface tension force downward around circle is twice the surface tension times the circumference, since two surfaces contribute to the force:

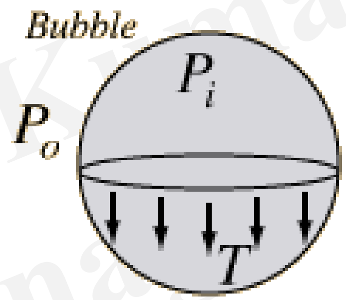
$$F_{downward} = 2T(2\pi r)$$

Surface Tension

- This gives

$$P_i - P_o = \frac{4T}{r} \quad \text{for a bubble}$$

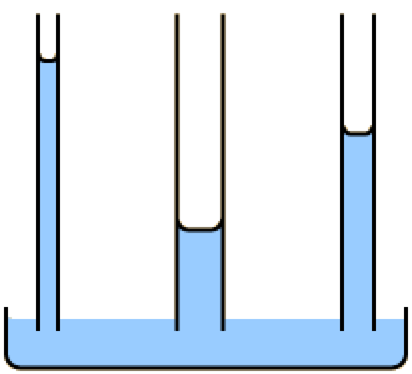
$$P_i - P_o = \frac{2T}{r} \quad \text{for a droplet which has only one surface.}$$

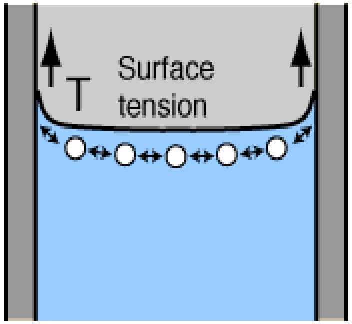


- This latter case also applies to the case of a bubble surrounded by a liquid

Capillarity

- Capillary action is the result of adhesion and surface tension. Adhesion of water to the walls of a vessel will cause an upward force on the liquid at the edges and result in a meniscus which turns upward. The surface tension acts to hold the surface intact, so instead of just the edges moving upward, the whole liquid surface is dragged upward.

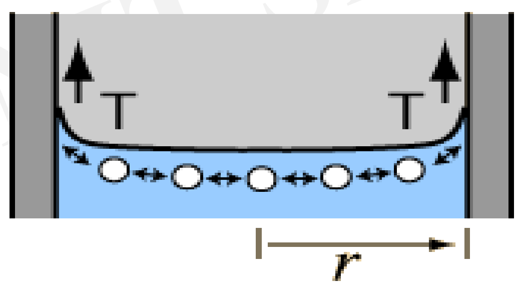




Capillarity

- Capillary action occurs when the adhesion to the walls is stronger than the cohesive forces between the liquid molecules. The height to which capillary action will take water in a uniform circular tube is limited by surface tension. Acting around the circumference, the upward force is

$$F_{upward} = T 2\pi r$$



T = surface tension

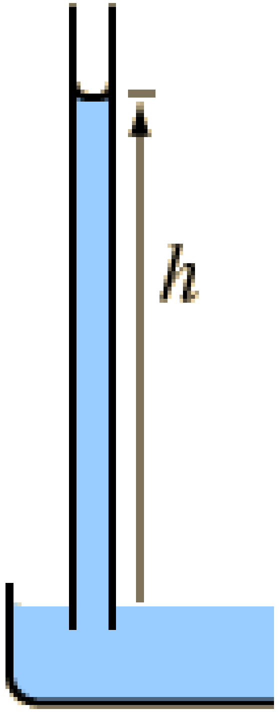
ρ = density of liquid

Capillarity

- The height h to which capillary action will lift water depends upon the weight of water which the surface tension will lift:

$$T2\pi r = \rho g(h\pi r^2)$$

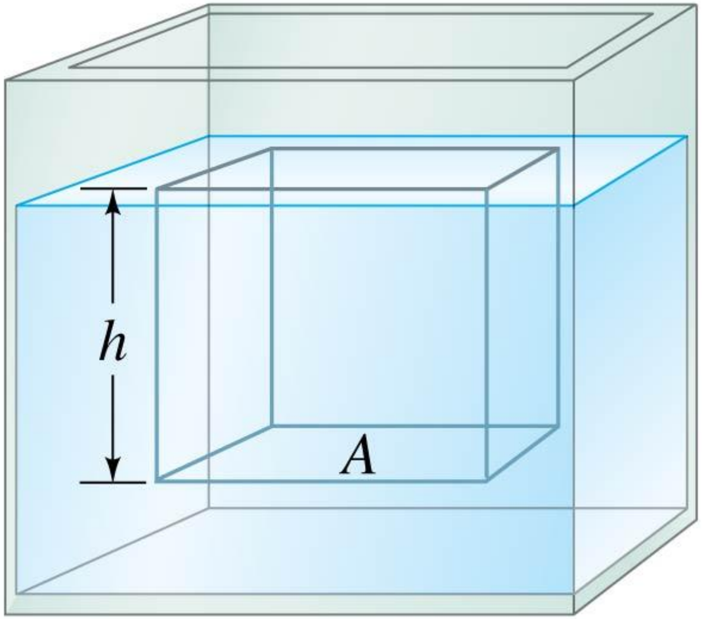
$$h = \frac{2T}{\rho r g}$$



Since it is weight limited it will rise higher in a smaller tube.

Pressure in Fluids

The pressure at a depth *h* below the surface of the liquid is due to the weight of the liquid above it. We can quickly calculate:



$$P = \frac{F}{A} = \frac{\rho A h g}{A}$$

$$P = \rho g h.$$

This relation is valid for any liquid whose density does not change with depth.

Atmospheric Pressure and Gauge Pressure

At sea level the atmospheric pressure is about 1.013×10^5 N/m^2; this is called one atmosphere (atm).

Another unit of pressure is the bar:

$$1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2$$

Standard atmospheric pressure is just over 1 bar.

This pressure does not crush us, as our cells maintain an internal pressure that balances it.

Atmospheric Pressure and Gauge Pressure

Most pressure gauges measure the pressure above the atmospheric pressure—this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_A + P_G$$

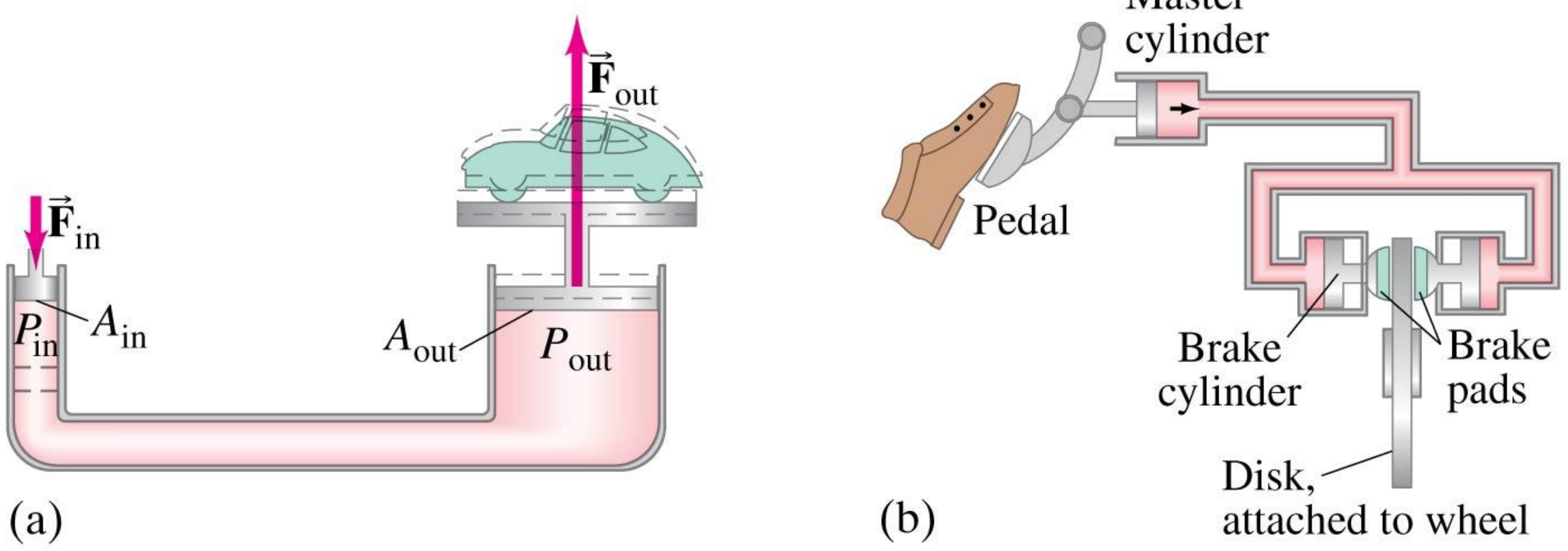
Hydrostatic Law

- The variation of pressure in vertical direction in a fluid is directly proportional to specific weight.
- $dp/dh = \rho g = w$
- $P = \rho g h$ (N/m^2)
- Note: When you move vertically down in a fluid, the pressure increases as $+\rho g h$.
- When you move vertically up in a fluid, the pressure decreases as $-\rho g h$.
- On the same horizontal level thee is no change of pressure.

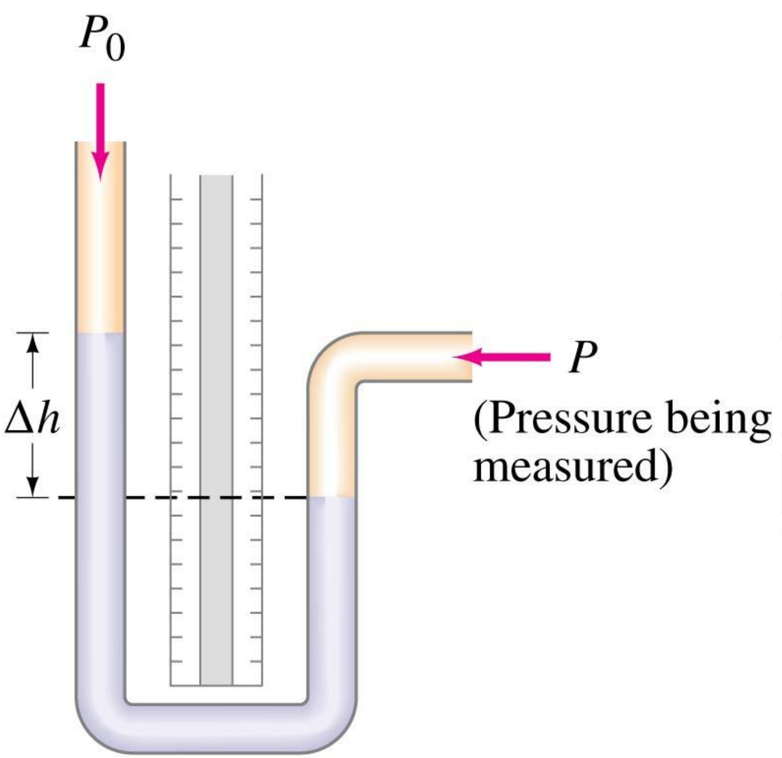
Pascal's Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

This principle is used, for example, in hydraulic lifts and hydraulic brakes.



Measurement of Pressure; Gauges and the Barometer

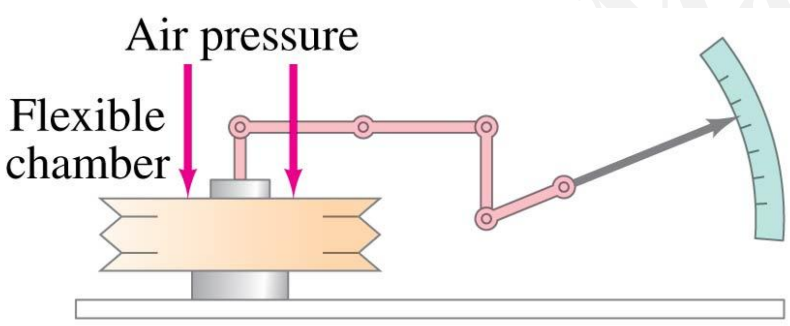


(a) Open-tube manometer

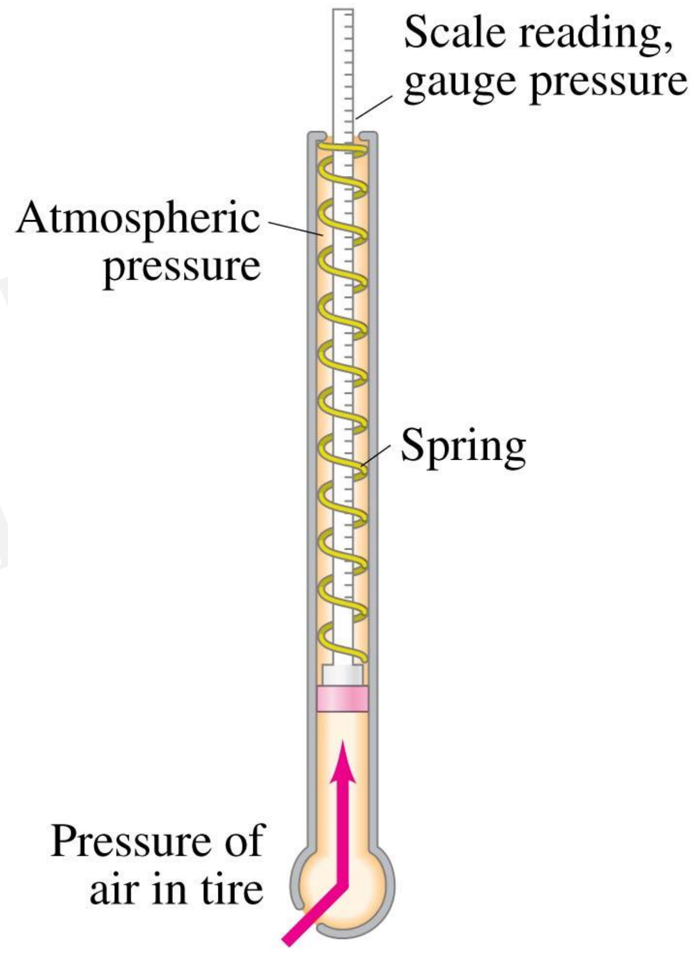
There are a number of different types of pressure gauges. This one is an open-tube manometer. The pressure in the open end is atmospheric pressure; the pressure being measured will cause the fluid to rise until the pressures on both sides at the same height are equal.

Measurement of Pressure; Gauges and the Barometer

Here are two more devices for measuring pressure: the aneroid gauge and the tire pressure gauge.

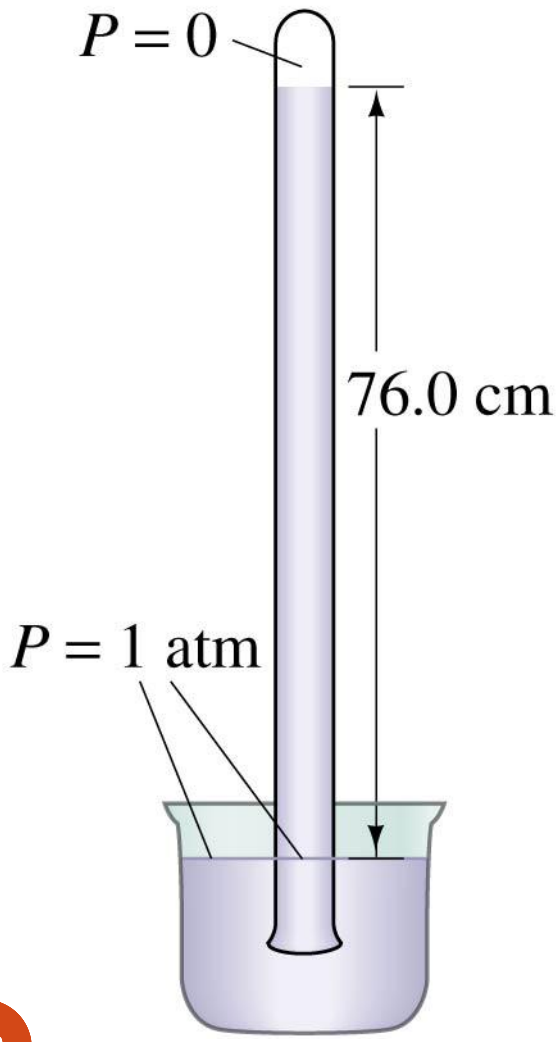


(b) Aneroid gauge (used mainly for air pressure, and then called an aneroid barometer)



(c) Tire gauge

Measurement of Pressure; Gauges and the Barometer

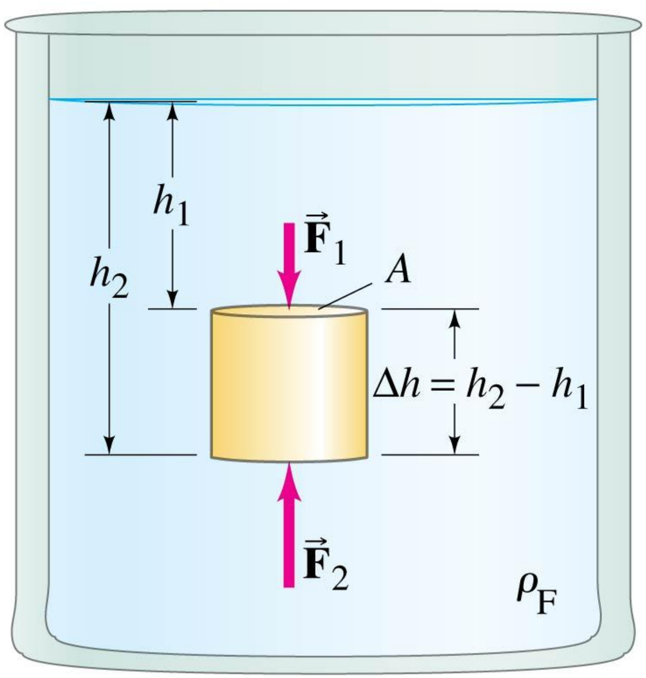


This is a mercury barometer, developed by Torricelli to measure atmospheric pressure. The height of the column of mercury is such that the pressure in the tube at the surface level is 1 atm.

Therefore, pressure is often quoted in millimeters (or inches) of mercury.

Buoyancy and Archimedes' Principle

This is an object submerged in a fluid. There is a net force on the object because the pressures at the top and bottom of it are different.

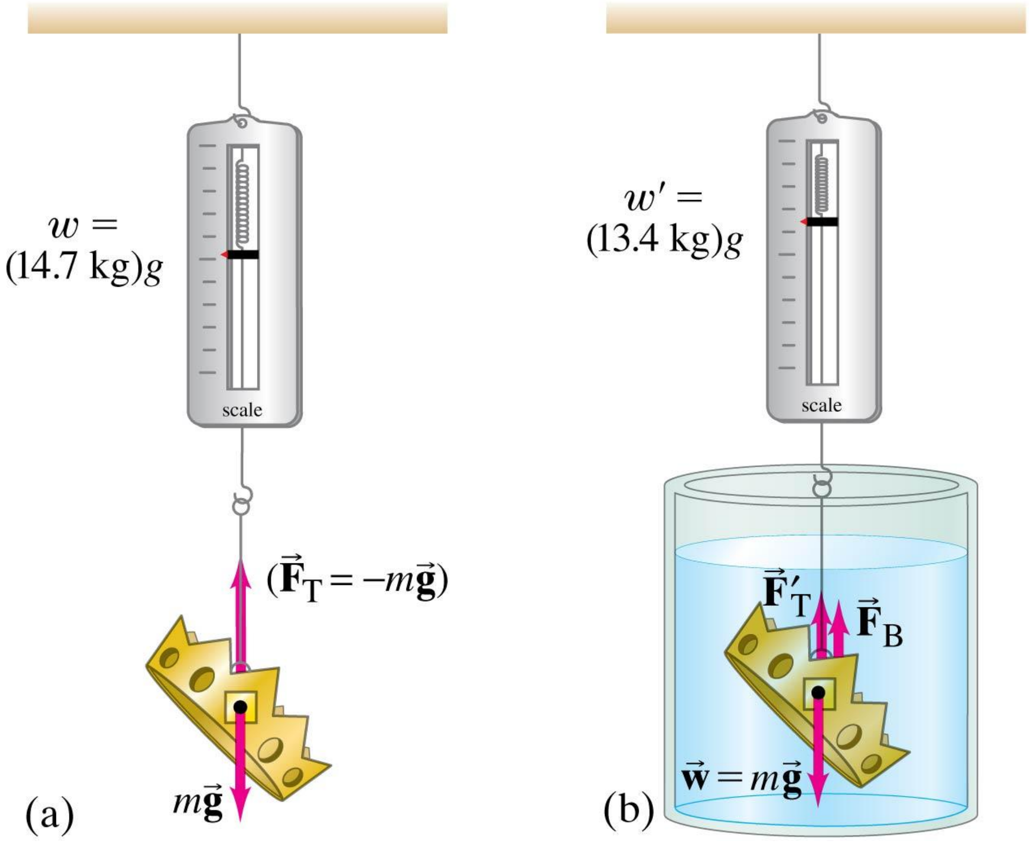


The buoyant force is found to be the upward force on the same volume of water:

$$
\begin{aligned}
F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\
&= \rho_F g A \, \Delta h \\
&= \rho_F V g \\
&= m_F g,
\end{aligned}
$$

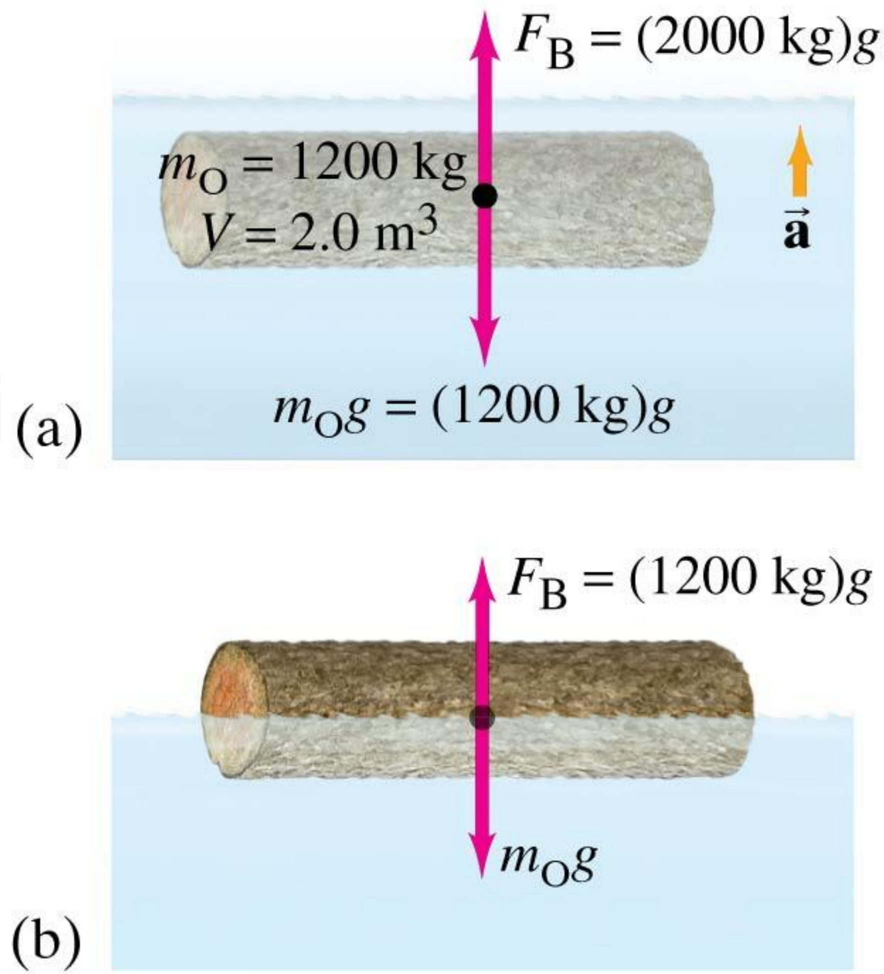
Buoyancy and Archimedes' Principle

The net force on the object is then the difference between the buoyant force and the gravitational force.



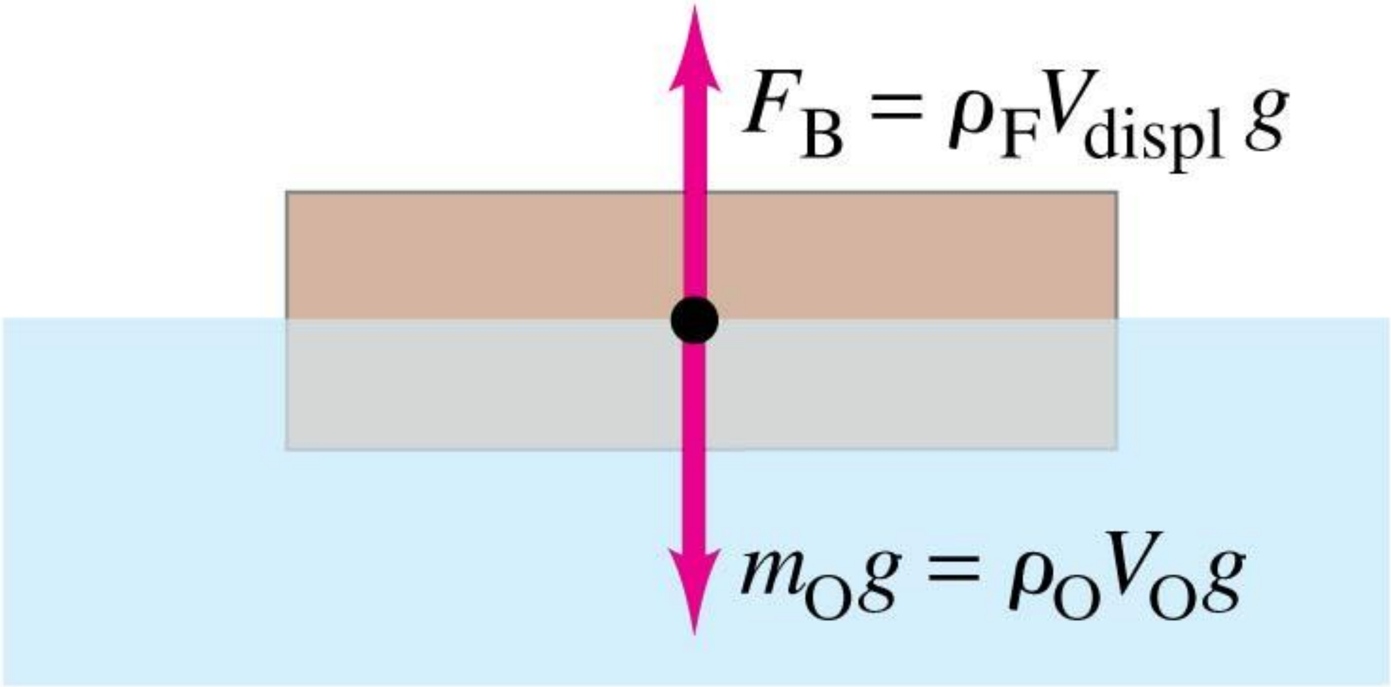
Buoyancy and Archimedes' Principle

If the object's density is less than that of water, there will be an upward net force on it, and it will rise until it is partially out of the water.

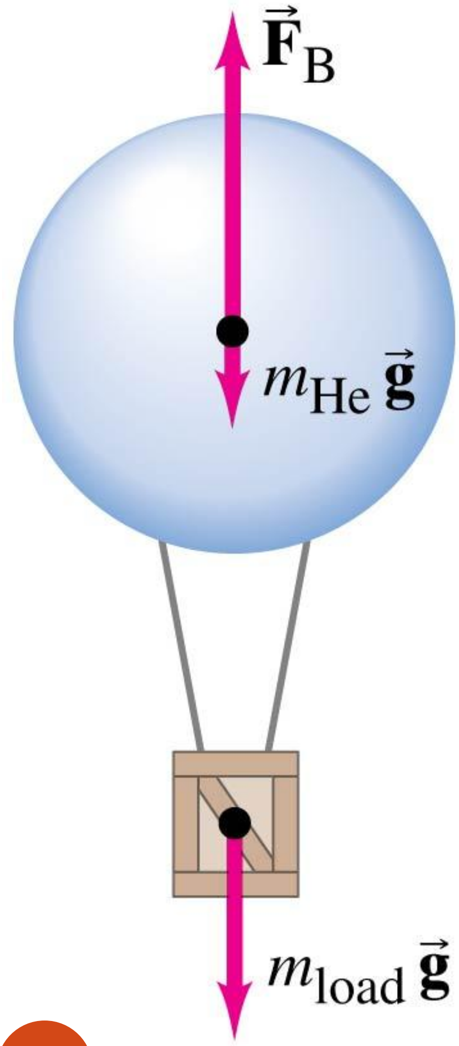


Buoyancy and Archimedes' Principle

For a floating object, the fraction that is submerged is given by the ratio of the object's density to that of the fluid.



Buoyancy and Archimedes' Principle



This principle also works in the air; this is why hot-air and helium balloons rise.

Types of Fluid Flow

- Steady and unsteady flow
  - Steady flow: flow in which fluid properties are not changing w.r.t. time but at given cross section.
  - Unsteady flow: flow in which fluid properties are changing w.r.t. time but at given cross section.
- Uniform and Non uniform flow
  - Uniform flow: Fluid is said to be in uniform flow if the velocity is not changing w.r.t. cross section but at a given interval of time.
  - Non- uniform flow: Fluid is said to be in uniform flow if the velocity is changing w.r.t. cross section but at a given interval of time.

Types of Fluid Flow

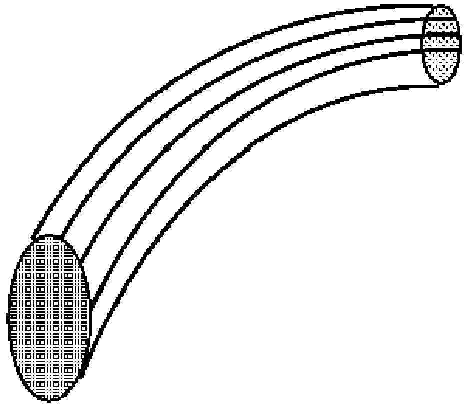
- Laminar and Turbulent flow
  - Laminar flow: A laminar flow is one in which fluid flow is in the form of layers and there is no intermixing of fluid particles or molecular momentum transfer.
  - Turbulent flow: A turbulent flow is one in which there is high order of intermixing of fluid particles.
- Rotational and irrotational flow
  - Rotational flow: If the fluid particles rotate about their axis or centre of mass.

Tools used to study fluid flow

- **Streamlines** are a family of curves that are instantaneously tangent to the velocity vector of the flow. These show the direction a fluid element will travel in at any point in time.
- **Streak lines** are the locus of points of all the fluid particles that have passed continuously through a particular spatial point in the past. Dye steadily injected into the fluid at a fixed point extends along a streak line.
- **Path lines** are the trajectories that individual fluid particles follow. These can be thought of as a "recording" of the path a fluid element in the flow takes over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time.

Stream tube

- A useful technique in fluid flow analysis is to consider only a part of the total fluid in isolation from the rest.
- This can be done by imagining a tubular surface formed by streamlines along which the fluid flows. This tubular surface is known as a *stream tube*.



A Streamtube

Stream tube

- The "walls" of a stream tube are made of streamlines.
- Fluid cannot flow across a streamline, so fluid cannot cross a stream tube wall.
- The stream tube can often be viewed as a solid walled pipe. A stream tube is **not** a pipe - it differs in unsteady flow as the walls will move with time.
- It differs because the "wall" is moving with the fluid

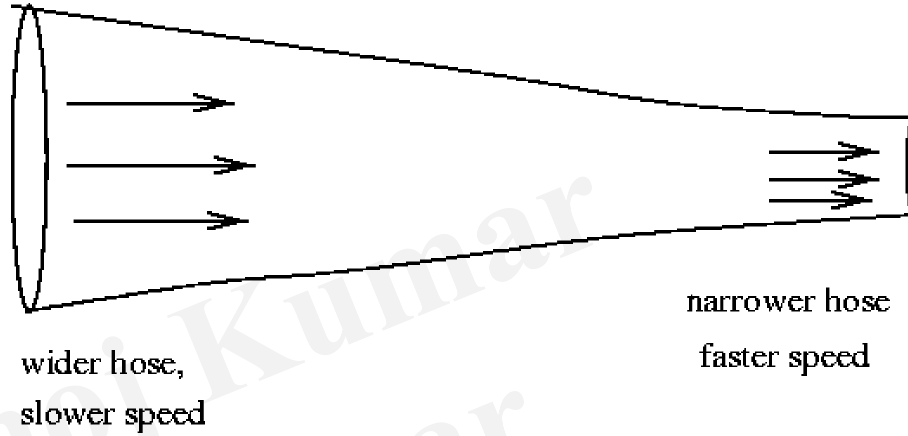
Generalized Continuity Equation

$$\frac{\partial \rho}{\partial t} = -\nabla \bullet (\rho \vec{V}) = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}$$

"Convergence of Density"

Change of density with respect to time

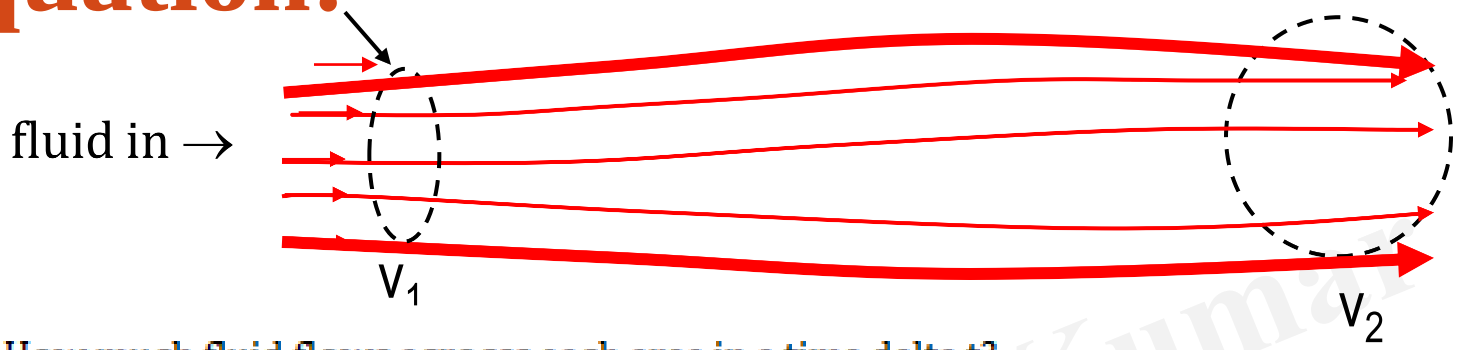
Conservation of mass: Continuity Equation:



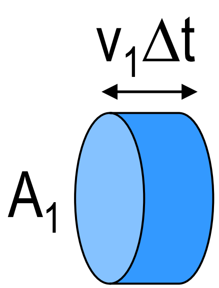
"The water all has to go somewhere"

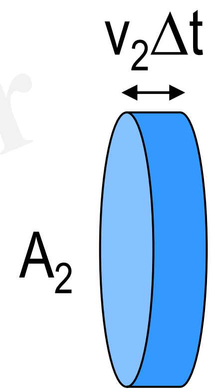
The rate a fluid enters a pipe must equal the rate the fluid leaves the pipe.
i.e. There can be **no sources or sinks** of fluid.

Conservation of mass: Continuity Equation:



How much fluid flows acrocss each area in a time delta t?





$$\Delta m = \rho V_1 = \rho A_1 v_1 \Delta t$$

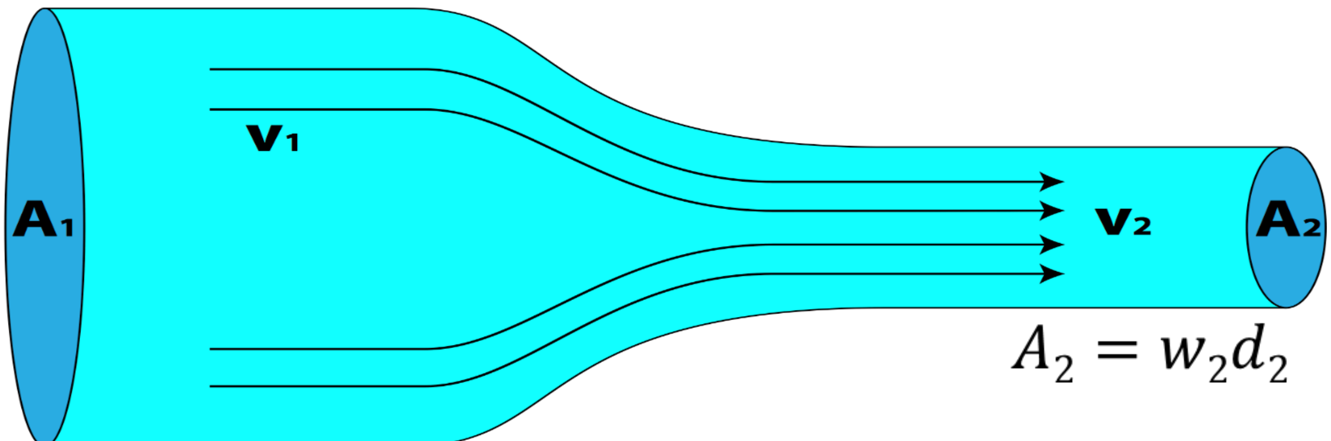
$$\Delta m = \rho V_2 = \rho A_2 v_2 \Delta t$$

$$flow\ rate: \frac{\Delta m}{\Delta t} = \rho A v$$

$$continuity\ eqn: \quad A_1 v_1 = A_2 v_2$$

Conservation of mass: Continuity Equation:

- Q. A river is 40m wide, 2.2m deep and flows at 4.5 m/s. It passes through a 3.7-m wide gorge, where the flow rate increases to 6.0 m/s. How deep is the gorge?



$$A_2 = w_2 d_2$$

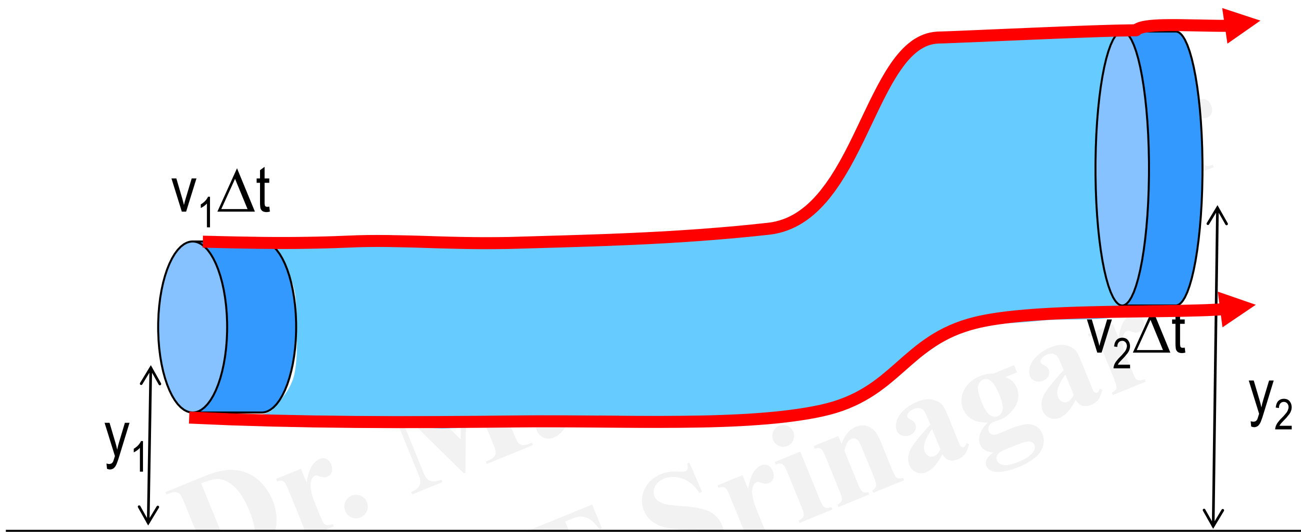
$$A_1 = w_1 d_1$$

$$Continuity\ equation: A_1 v_1 = A_2 v_2 \rightarrow w_1 d_1 v_1 = w_2 d_2 v_2$$

$$d_2 = \frac{w_1 d_1 v_1}{w_2 v_2} = \frac{40 \times 2.2 \times 4.5}{3.7 \times 6.0} = 18\ m$$

Conservation of mass: Continuity Equation:

What happens to the energy density of the fluid if I raise the ends ?



Energy per unit volume

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2 = const$$

Total energy per unit volume is constant at any point in fluid.

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = const$$

Conservation of mass: Continuity Equation:

- Q. Find the velocity of water leaving a tank through a hole in the side 1 metre below the water level.

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

$$At\ the\ top\text{:}\ P = 1\ atm, v = 0, y = 1\ m$$

$$At\ the\ bottom\text{:}\ P = 1\ atm, v = ?, y = 0\ m$$

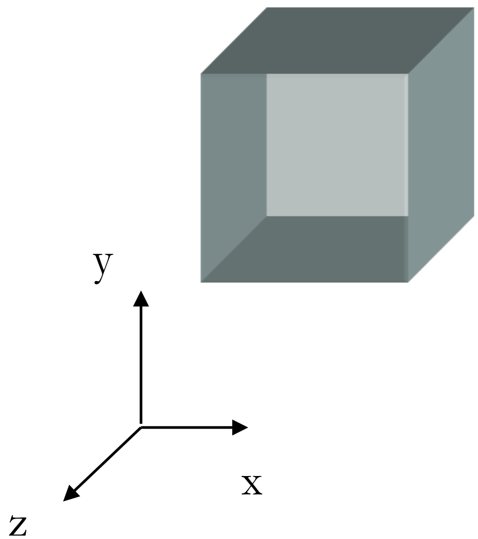
$$\cancel{P} + \rho g y = \cancel{P} + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gy} = \sqrt{2 \times 9.8 \times 1} = 4.4\ m/s$$

Momentum Conservation Equation

From Newton's second law : Force = (mass)(acceleration)

Consider a small element $\delta x \delta y \delta z$ as shown below.

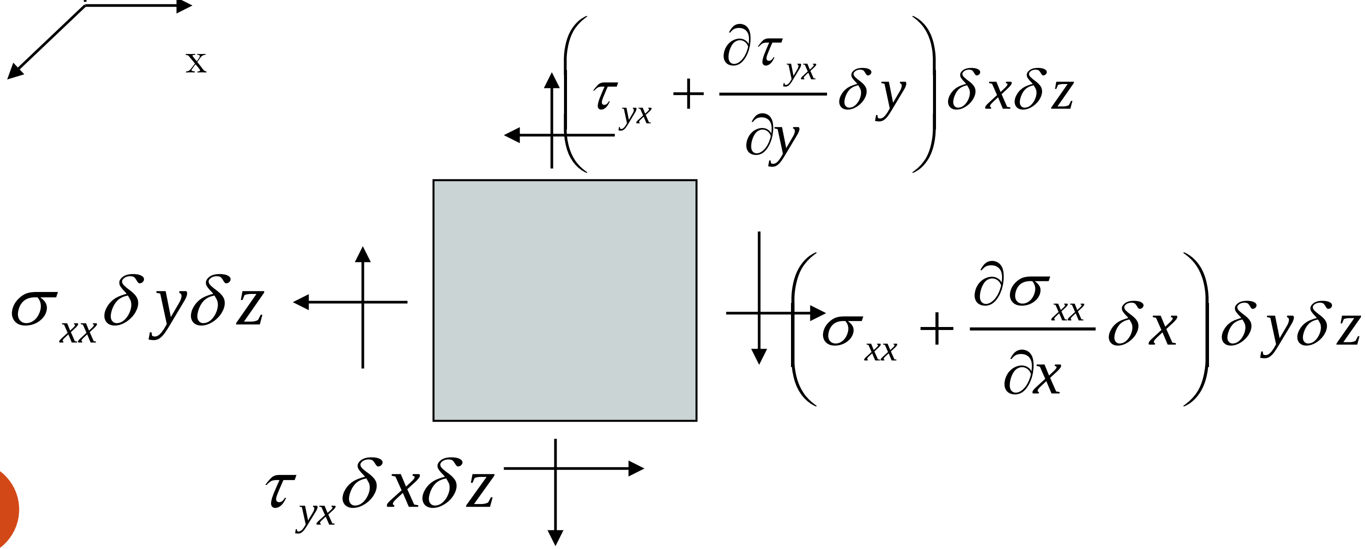


The element experiences an acceleration

$$m\frac{D\vec{V}}{Dt} = \rho(\delta x \delta y \delta z)\left(\frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}\right)$$

as it is under the action of various forces:

normal stresses, shear stresses, and gravitational force.



Momentum Balance (cont.)

Net force acting along the x-direction:

$$\frac{\partial \sigma_{\mathrm{xx}}}{\partial \mathrm{x}}\delta x \delta y \delta z + \frac{\partial \tau_{\mathrm{yx}}}{\partial \mathrm{x}}\delta x \delta y \delta z + \frac{\partial \tau_{\mathrm{zx}}}{\partial \mathrm{x}}\delta x \delta y \delta z + \rho g_x \delta x \delta y \delta z$$

Normal stress

Shear stresses (note: τ_{zx}: shear stress acting on surfaces perpendicular to the z-axis, not shown in previous slide)

Body force

The differential momentum equation along the x-direction is

$$\frac{\partial \sigma_{\mathrm{xx}}}{\partial \mathrm{x}} + \frac{\partial \tau_{\mathrm{yx}}}{\partial \mathrm{x}} + \frac{\partial \tau_{\mathrm{zx}}}{\partial \mathrm{x}} + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

similar equations can be derived along the y & z directions

Euler's Equations

For an inviscid flow, the shear stresses are zero and the normal stresses are simply the pressure: $\tau = 0$ for all shear stresses, $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -P$

$$-\frac{\partial P}{\partial x} + \rho g_x = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

Similar equations for y & z directions can be derived

$$-\frac{\partial P}{\partial y} + \rho g_y = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$-\frac{\partial P}{\partial z} + \rho g_z = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

Note: Integration of the Euler's equations along a streamline will give rise to the Bernoulli's equation.

Navier and Stokes Equations

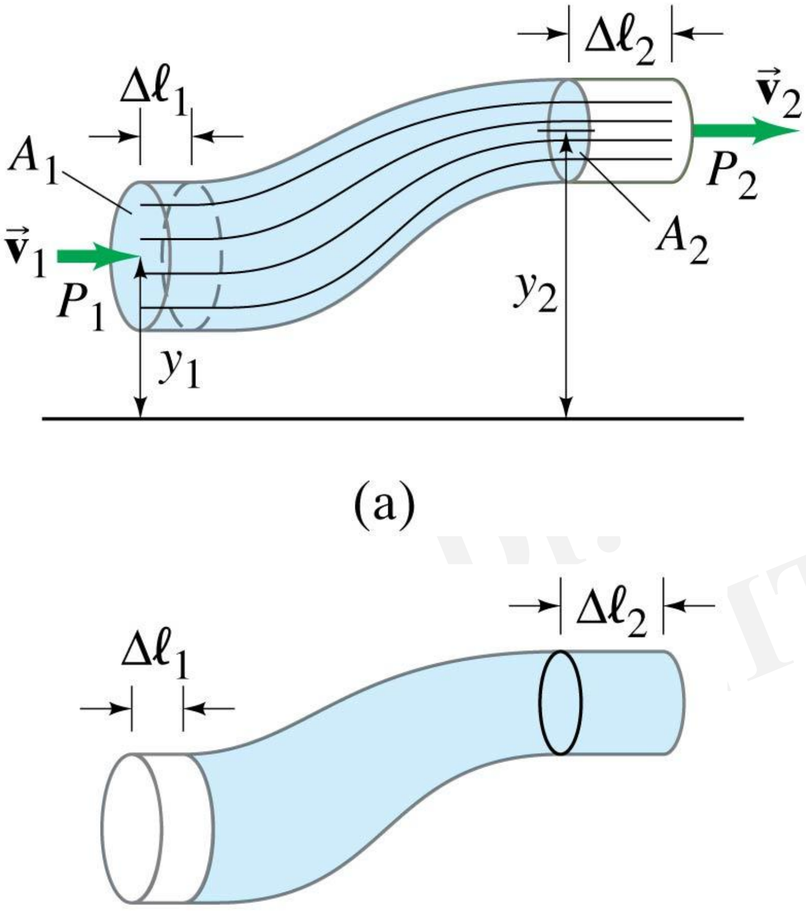
For a viscous flow, the relationships between the normal/shear stresses and the rate of deformation (velocity field variation) can be determined by making a simple assumption. That is, the stresses are linearly related to the rate of deformation (Newtonian fluid). The proportional constant for the relation is the dynamic viscosity of the fluid (μ). Based on this, Navier and Stokes derived the famous Navier-Stokes equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Bernoulli’s Equation



(a)
(b)

A fluid can also change its height. By looking at the work done as it moves, we find:

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 .$$

This is Bernoulli’s equation. One thing it tells us is that as the speed goes up, the pressure goes down.

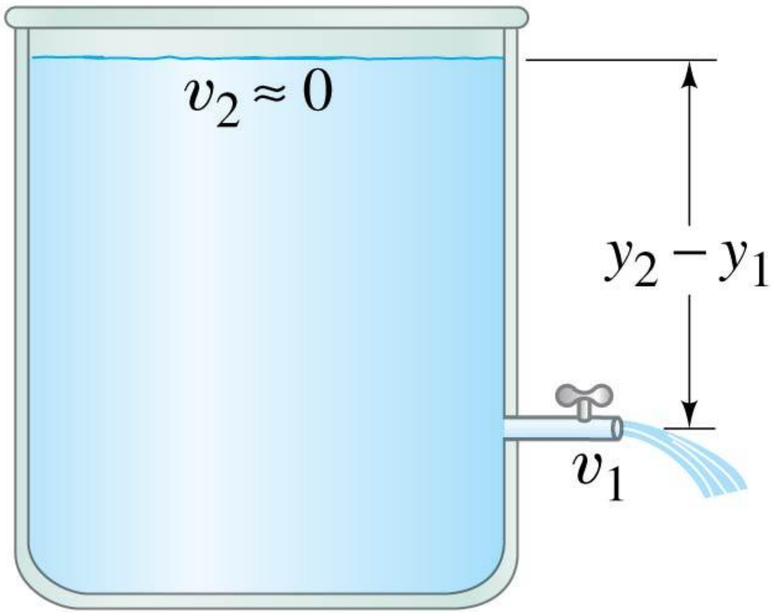
Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow

Using Bernoulli's principle, we find that the speed of fluid coming from a spigot on an open tank is:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

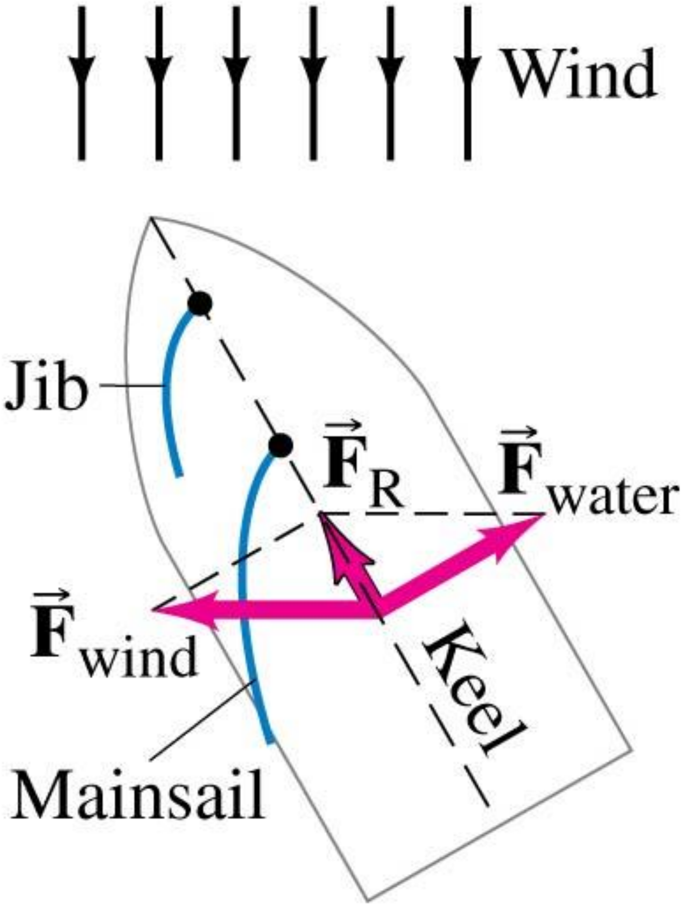
or

$$v_1 = \sqrt{2g(y_2 - y_1)}.$$



This is called Torricelli's theorem.

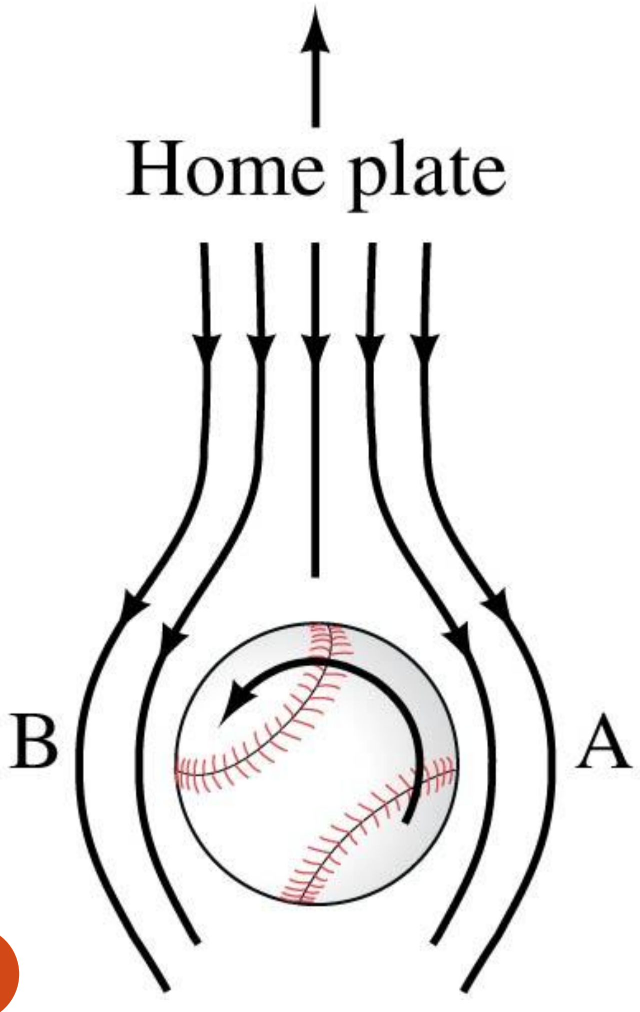
Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow



(b)

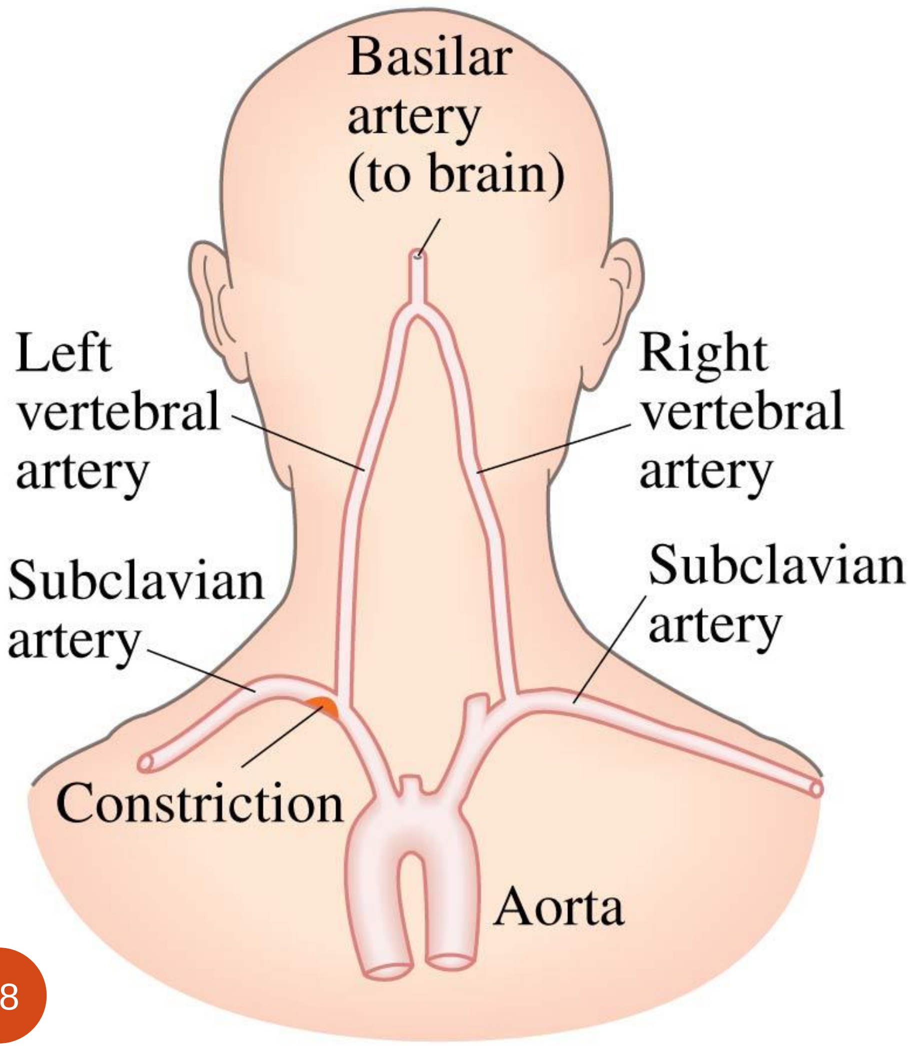
A sailboat can move against the wind, using the pressure differences on each side of the sail, and using the keel to keep from going sideways.

Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow



A ball's path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal.

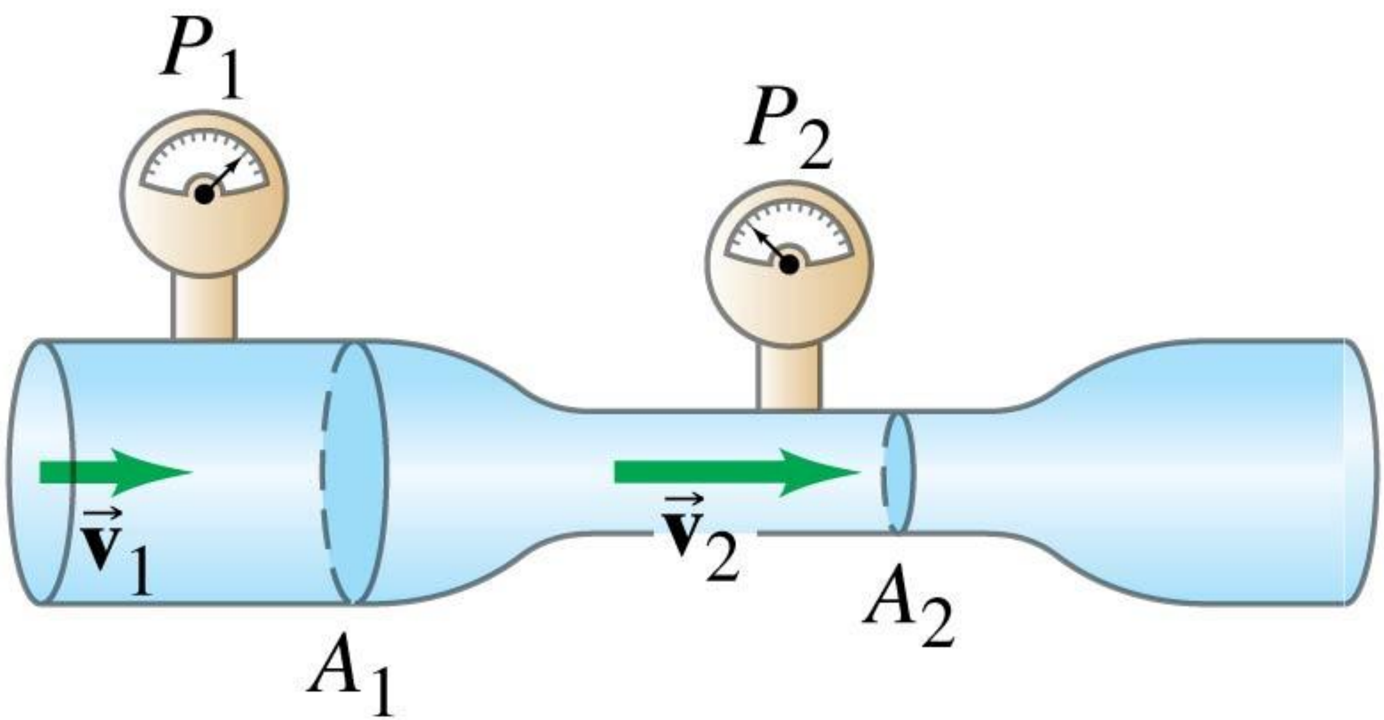
Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow



A person with constricted arteries will find that they may experience a temporary lack of blood to the brain as blood speeds up to get past the constriction, thereby reducing the pressure.

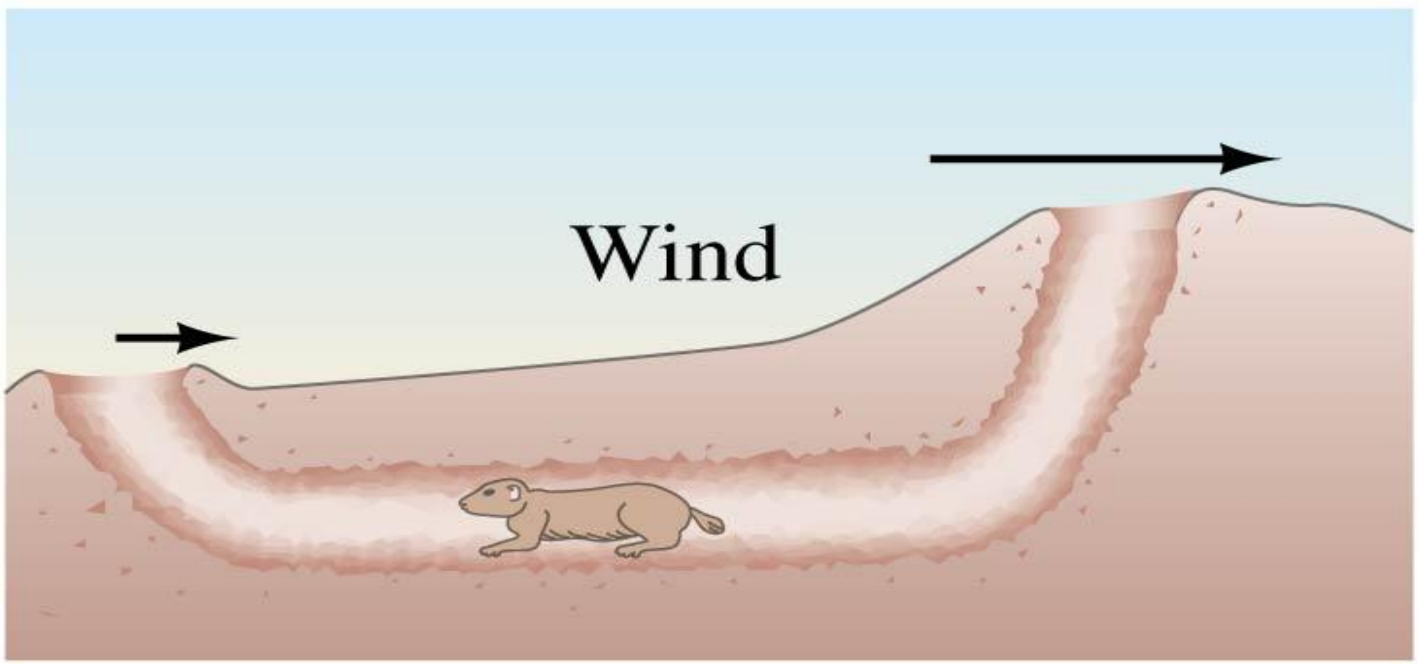
Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow

A venturi meter can be used to measure fluid flow by measuring pressure differences.



Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow

Air flow across the top helps smoke go up a chimney, and air flow over multiple openings can provide the needed circulation in underground burrows.



Flow in Tubes; Poiseuille’s Equation, Blood Flow

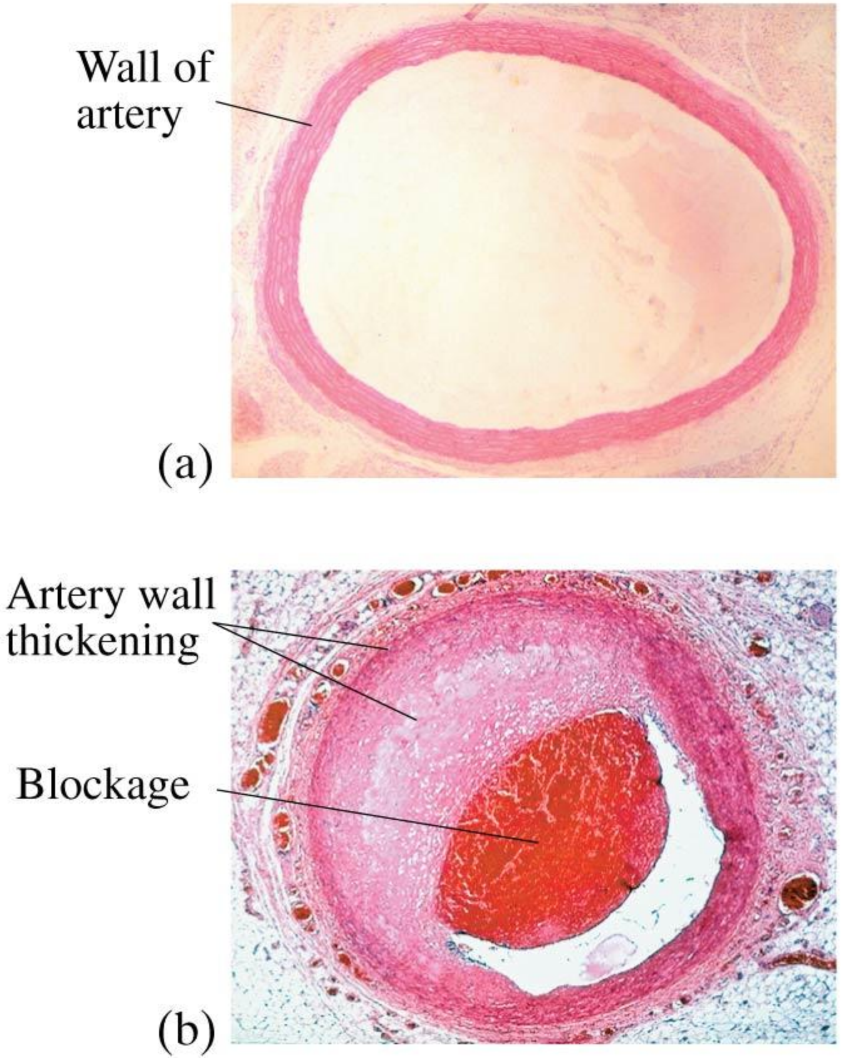
The rate of flow in a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube.

The volume flow rate is proportional to the pressure difference, inversely proportional to the length of the tube and to the pressure difference, and proportional to the fourth power of the radius of the tube.

Flow in Tubes; Poiseuille's Equation, Blood Flow

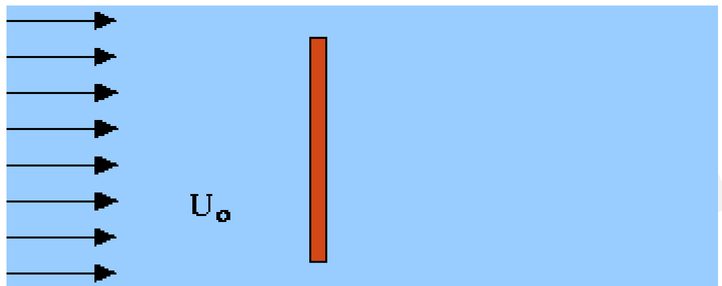
This has consequences for blood flow—if the radius of the artery is half what it should be, the pressure has to increase by a factor of 16 to keep the same flow.

Usually the heart cannot work that hard, but blood pressure goes up as it tries.

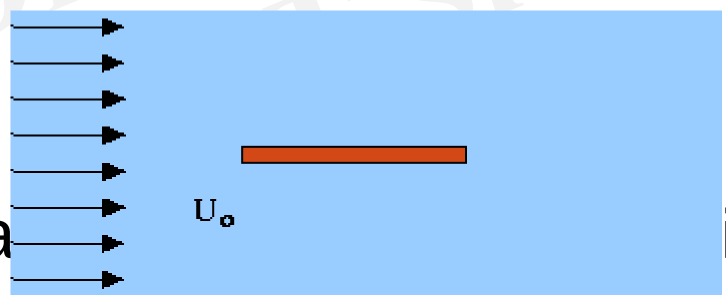


Drag on a surface – 2 types

- Pressure stress / form drag



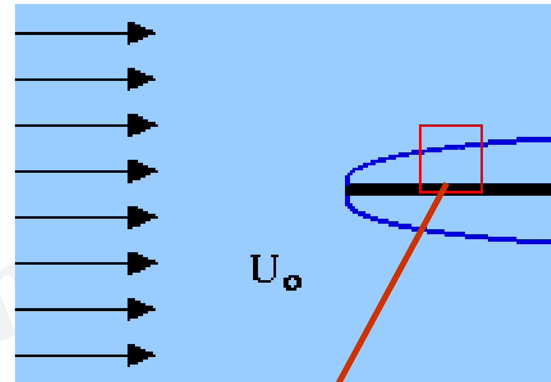
- Shear stress / skin friction drag

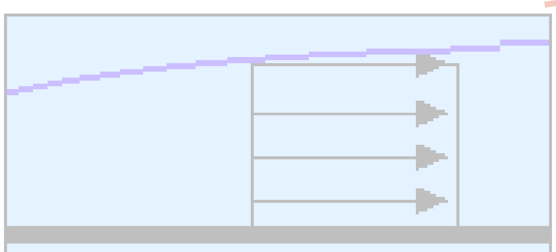


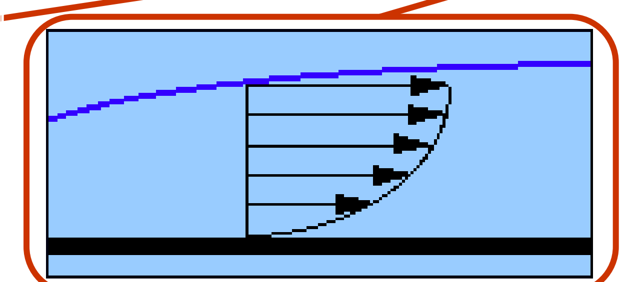
- A boundary la... iction

Boundary layer – velocity profile

- Far from the surface, the fluid velocity is unaffected.
- In a thin region near the surface, the velocity is reduced
- Which is the "most correct" velocity profile?

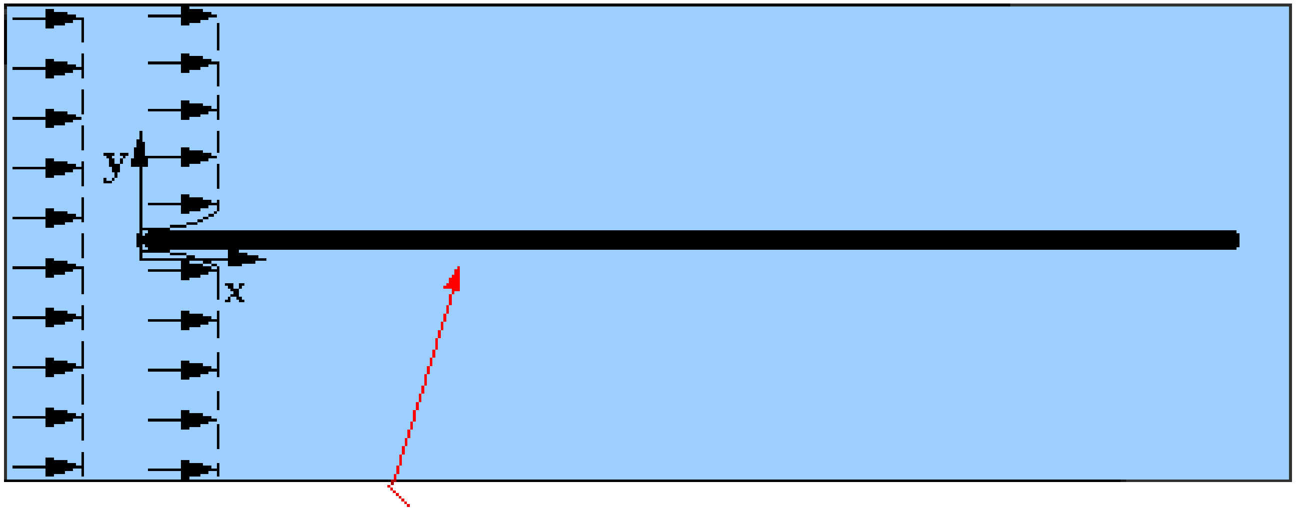






…this is a good approximation near the "front" of the plate

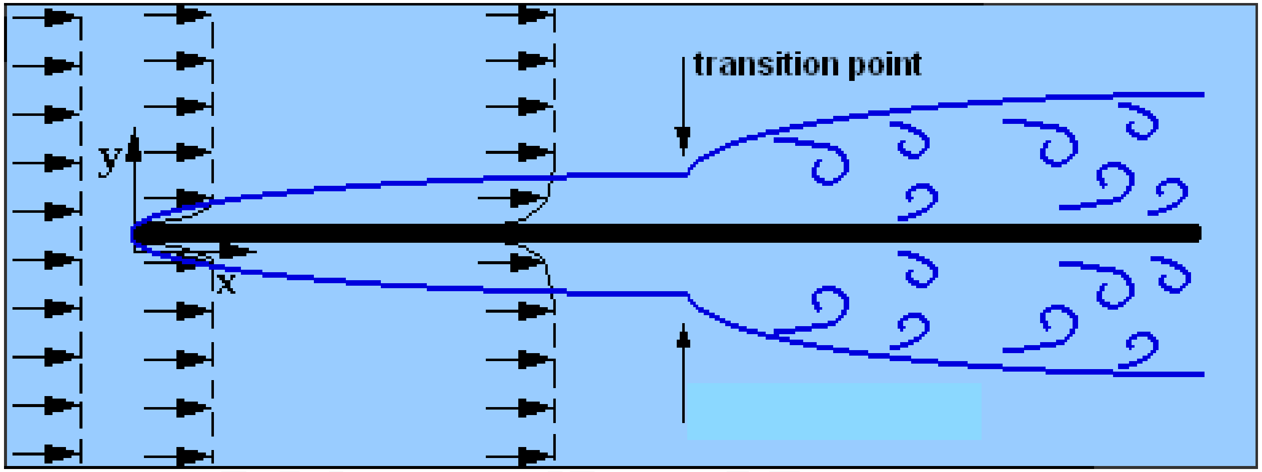
Boundary layer growth



- The free stream velocity is u0, but next to the plate, the flow is reduced by drag
- Farther along the plate, the affect of the drag is felt by more of the stream, and because of this
- The boundary layer grows

Boundary layer transition

- At a certain point, viscous forces become to small relative to inertial forces to damp fluctuations

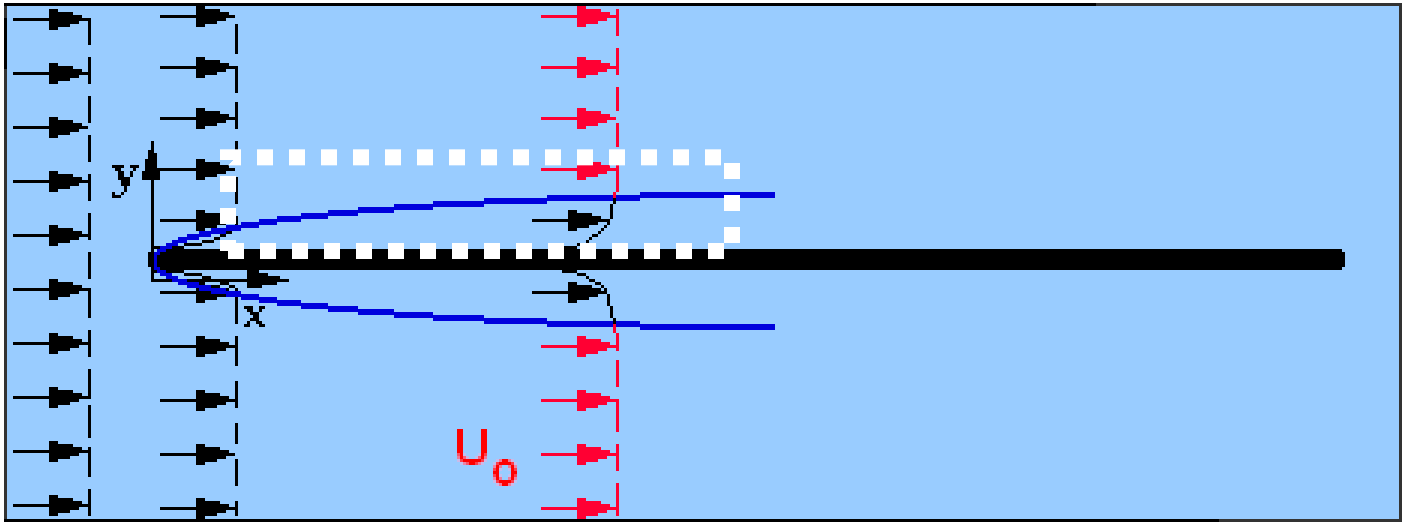


- The flow transitions to turbulence
- Important parameters:
  - Viscosity μ, density ρ
  - Distance, x
  - Velocity U_O
- Reynolds number combines these into one number

$$Re_x = \frac{\rho U_O x}{\mu} = \frac{U_O x}{\nu}$$

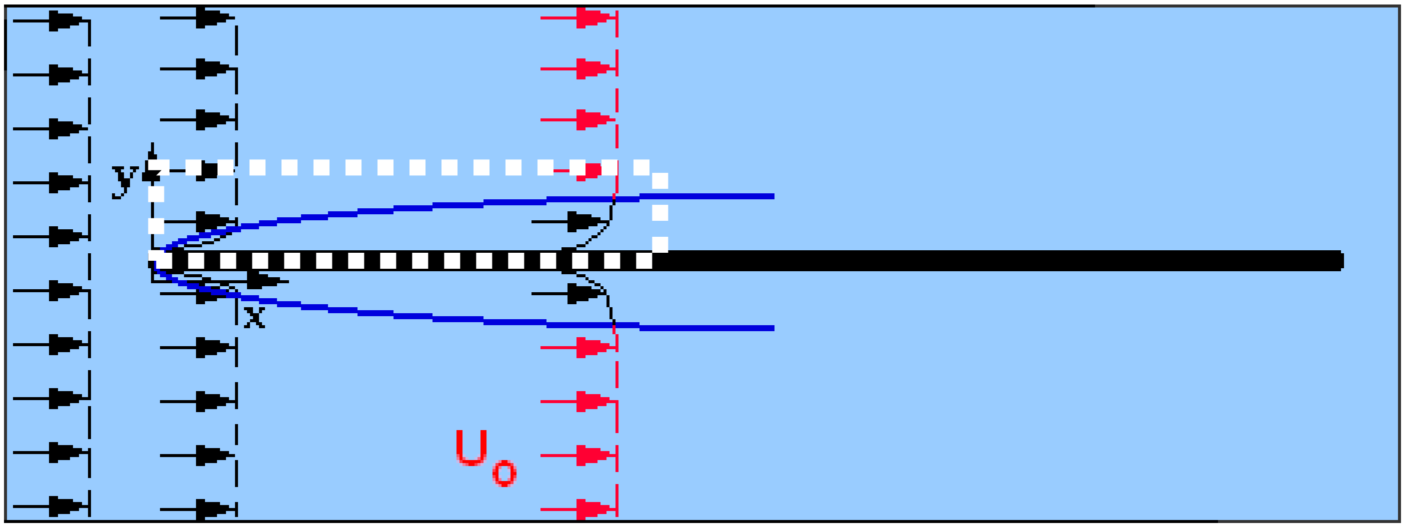
First focus on “laminar” boundary layer

- A practical “outer edge” of the boundary layer is where $u = u_o \times 99\%$

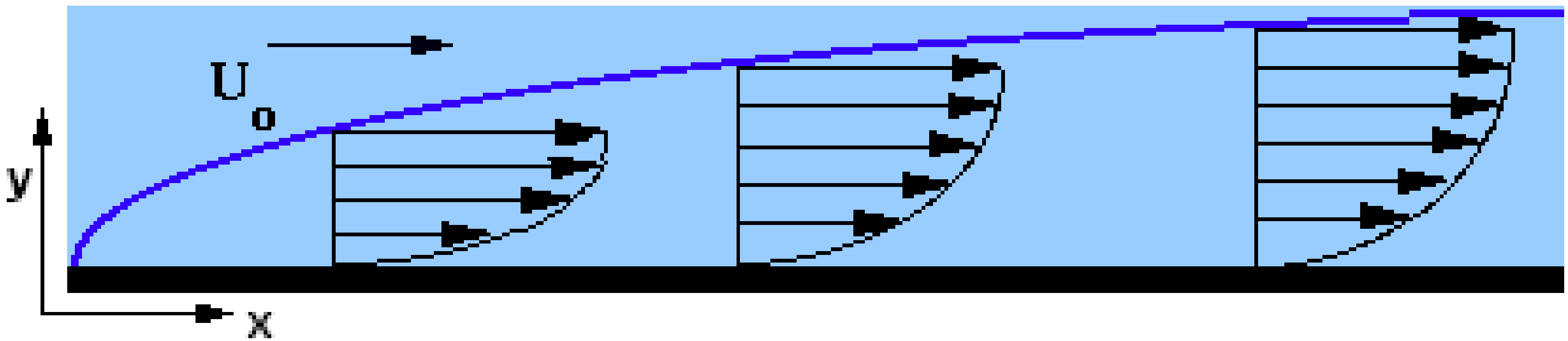


- Across the boundary layer there is a velocity gradient du/dy that we will use to determine τ

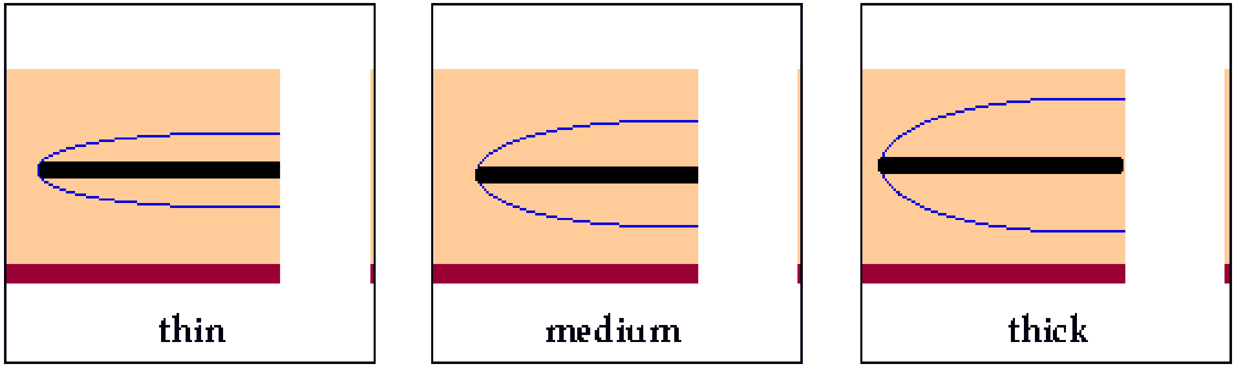
- Let's look at the growth of the boundary layer quantitatively.



- The velocity profiles grow along the surface



- What determines the growth rate and flow profile?



Laminar Flat-Plate Boundary Layer: Exact Solution

- Governing Equations $\nabla \cdot \vec{V} = 0$

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho \left(\vec{V} \cdot \nabla \right) \vec{V} = -\nabla p + \mu \nabla^2 \vec{V} + \rho \vec{g}$$

- For incompressible steady 2D cases:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Laminar Flat-Plate Boundary Layer: Exact Solution

- Boundary Conditions

$$y = 0, \qquad u = 0, \qquad v = 0$$

$$y = \infty, \qquad u = U, \qquad \frac{\partial u}{\partial y} = 0$$

- Equations are Coupled, Nonlinear, Partial Differential Equations
- Blassius Solution:
  – Transform to single, higher-order, nonlinear, ordinary differential equation

Laminar Flat-Plate Boundary Layer: Exact Solution

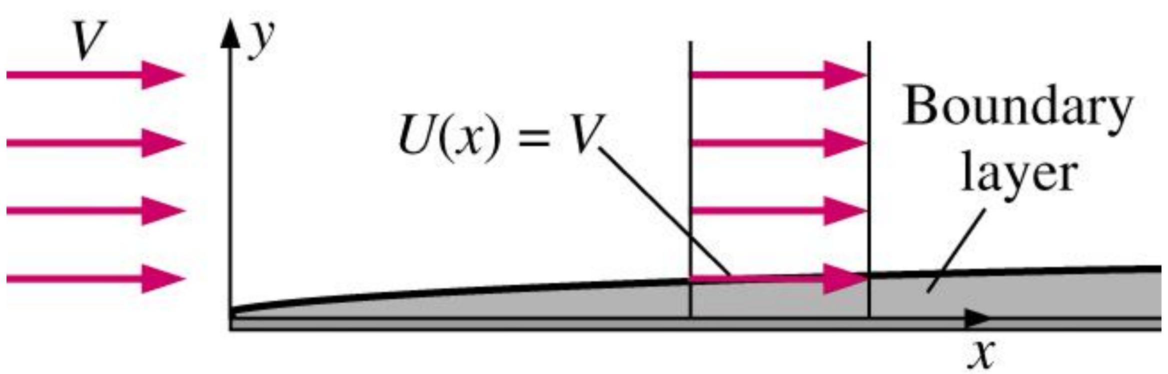
$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

$$\eta = 0, \qquad f = \frac{df}{d\eta} = 0$$

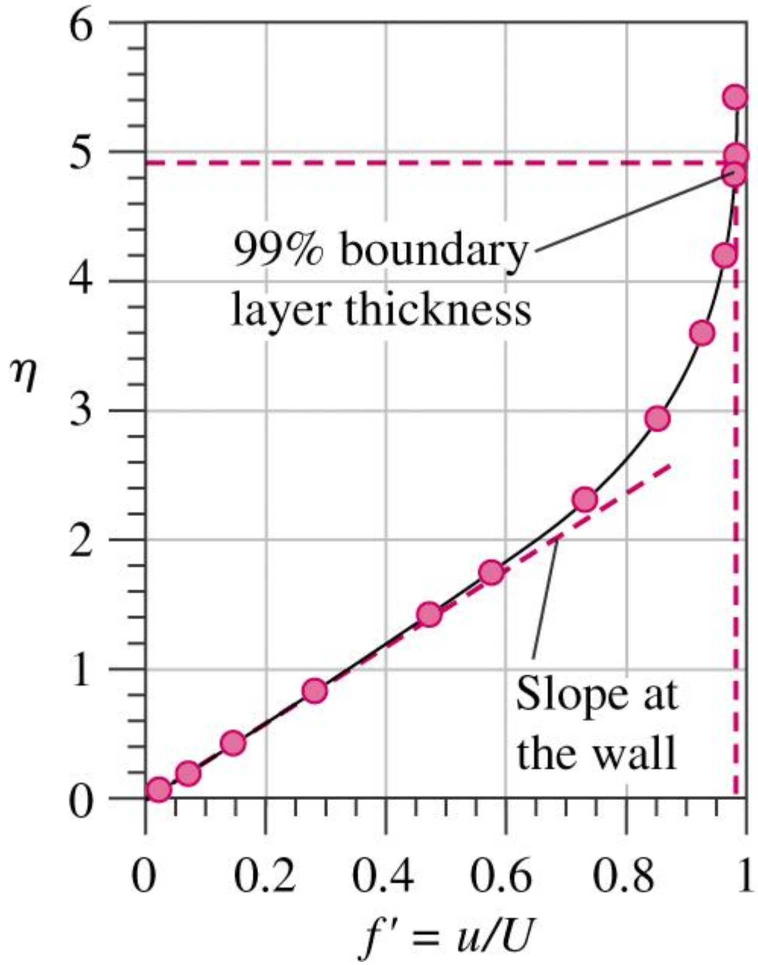
$$\eta \to \infty, \qquad \frac{df}{d\eta} = 1$$

Boundary Layer Procedure

- Before defining and δ^* and θ, are there analytical solutions to the BL equations?
  - *Unfortunately, NO*
- *Blasius Similarity Solution* boundary layer on a flat plate, constant edge velocity, zero external pressure gradient



Blasius Similarity Solution



- Blasius introduced similarity variables

$$f' = \frac{U}{U_e} \qquad \eta = y\sqrt{\frac{U_e}{\nu x}}$$

- This reduces the BLE to

$$f''' + f f'' = 0$$
$$f(0) = f'(0) = 0, \quad f'(\infty) = 1$$

- This ODE can be solved using Runge-Kutta technique
- Result is a BL profile which holds at every station along the flat plate

Blasius Similarity Solution

TABLE 10–3

Solution of the Blasius laminar flat plate boundary layer in similarity variables*

η	f''	f'	f	η	f''	f'	f
0.0	0.33206	0.00000	0.00000	2.4	0.22809	0.72898	0.92229
0.1	0.33205	0.03321	0.00166	2.6	0.20645	0.77245	1.07250
0.2	0.33198	0.06641	0.00664	2.8	0.18401	0.81151	1.23098
0.3	0.33181	0.09960	0.01494	3.0	0.16136	0.84604	1.39681
0.4	0.33147	0.13276	0.02656	3.5	0.10777	0.91304	1.83770
0.5	0.33091	0.16589	0.04149	4.0	0.06423	0.95552	2.30574
0.6	0.33008	0.19894	0.05973	4.5	0.03398	0.97951	2.79013
0.8	0.32739	0.26471	0.10611	5.0	0.01591	0.99154	3.28327
1.0	0.32301	0.32978	0.16557	5.5	0.00658	0.99688	3.78057
1.2	0.31659	0.39378	0.23795	6.0	0.00240	0.99897	4.27962
1.4	0.30787	0.45626	0.32298	6.5	0.00077	0.99970	4.77932
1.6	0.29666	0.51676	0.42032	7.0	0.00022	0.99992	5.27923
1.8	0.28293	0.57476	0.52952	8.0	0.00001	1.00000	6.27921
2.0	0.26675	0.62977	0.65002	9.0	0.00000	1.00000	7.27921
2.2	0.24835	0.68131	0.78119	10.0	0.00000	1.00000	8.27921

* η is the similarity variable defined in Eq. 4 above, and function $f(\eta)$ is solved using the Runge–Kutta numerical technique. Note that f'' is proportional to the shear stress τ, f' is proportional to the x-component of velocity in the boundary layer ($f' = u/U$), and f itself is proportional to the stream function. f' is plotted as a function of η in Fig. 10–99.

Blasius Similarity Solution

- Boundary layer thickness can be computed by assuming that δ corresponds to point where $U/U_e = 0.990$. At this point, $\eta = 4.91$, therefore

Recall

$$\eta = 4.91 = \sqrt{\frac{U_e}{\nu x}}\delta \;\Rightarrow\; \frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}} \qquad Re_x = \frac{\rho U x}{\mu}$$

- Wall shear stress τ_w and friction coefficient $C_{f,x}$ can be directly related to Blasius solution

$$\tau_w = \mu \left.\frac{\partial U}{\partial y}\right|_{y=0} = f''(0)\frac{\rho U_e^2}{\sqrt{Re_x}} = 0.332\frac{\rho U_e^2}{\sqrt{Re_x}} \qquad C_{f,x} = \frac{\tau_w}{\frac{1}{2}\rho U_e^2} = \frac{0.664}{\sqrt{Re_x}}$$

Displacement Thickness

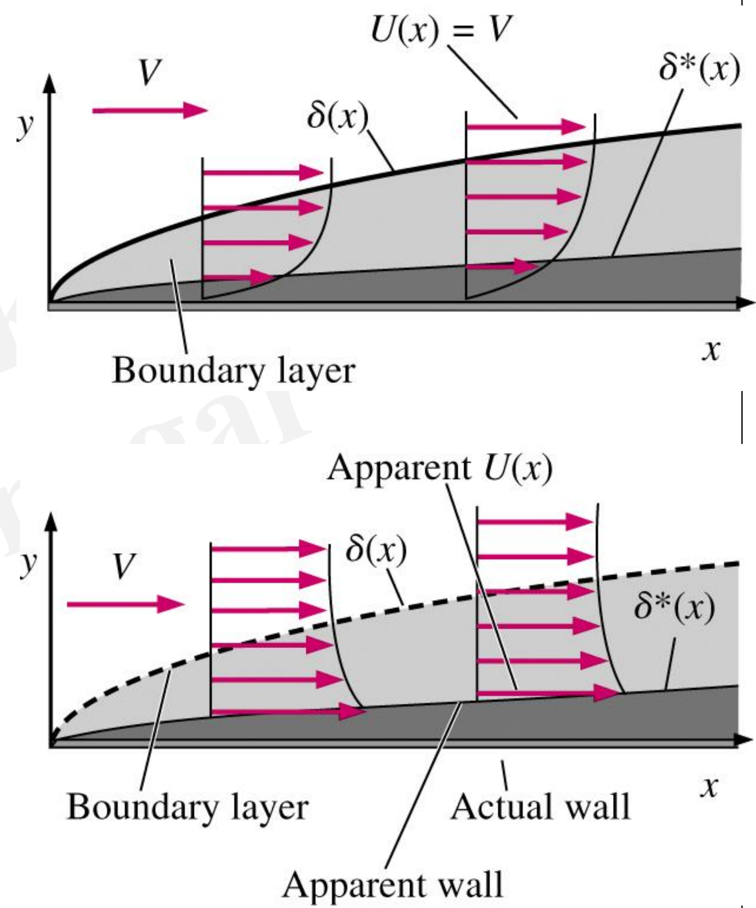
- Displacement thickness δ^* is the imaginary increase in thickness of the wall (or body), as seen by the outer flow, and is due to the effect of a growing BL.
- Expression for δ^* is based upon control volume analysis of conservation of mass

$$\delta^\star = \int_0^\infty \left(1 - \frac{U}{U_e}\right) \mathrm{d}y$$

- Blasius profile for laminar BL can be integrated to give

$$\frac{\delta^\star}{x} = \frac{1.72}{\sqrt{Re_x}}$$

(≈1/3 of δ)



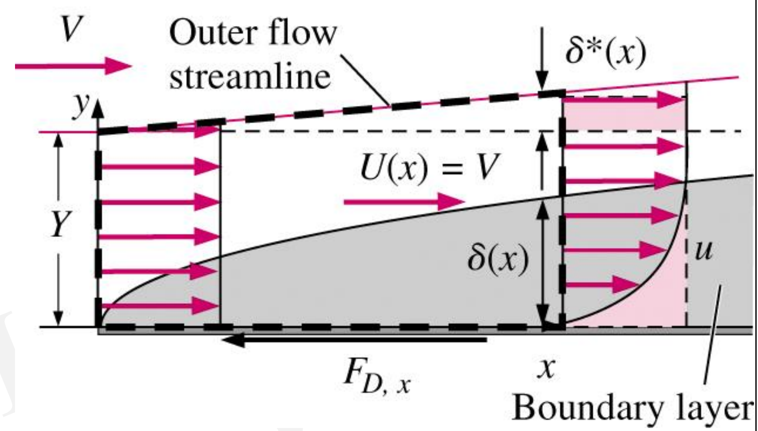
Momentum Thickness

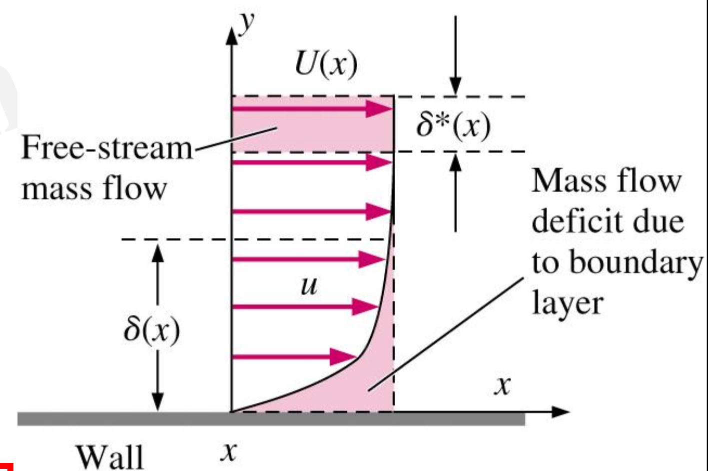
- Momentum thickness θ is another measure of boundary layer thickness.
- Defined as the loss of momentum flux per unit width divided by ρU^2 due to the presence of the growing BL.
- Derived using CV analysis.

$$\theta = \int_0^\infty \frac{U}{U_e}\left(1 - \frac{U}{U_e}\right) \mathrm{d}y = \frac{F_{D,x}}{\rho U_e^2 w}$$

$$\frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}}$$

θ for Blasius solution, identical to $C_{f,x}$





Turbulent Boundary Layer

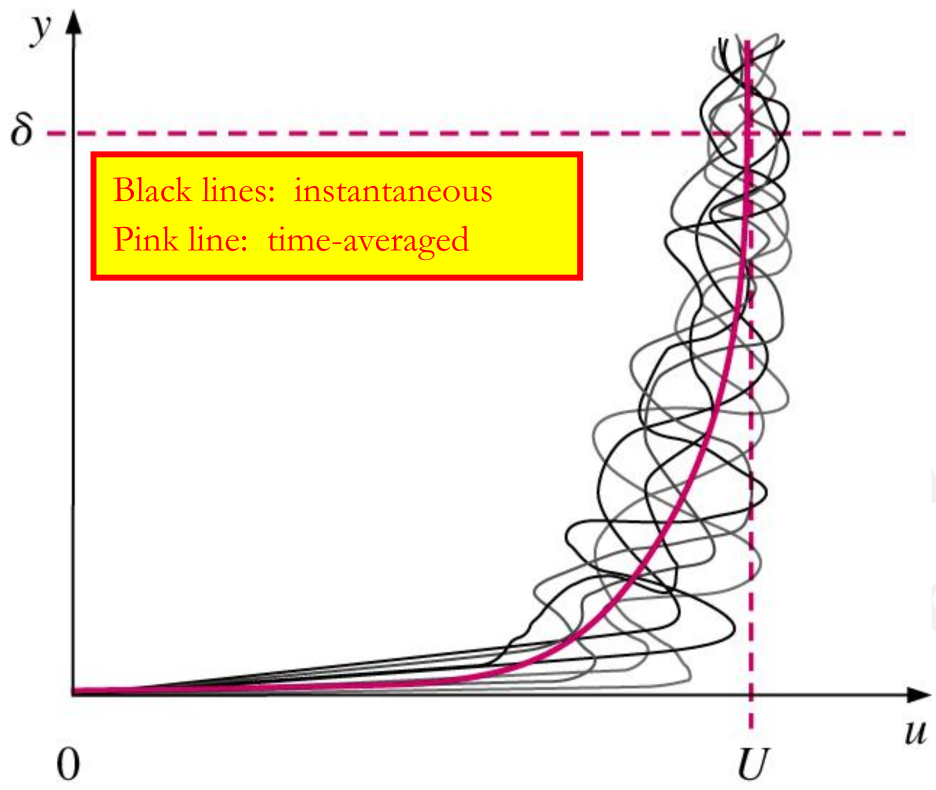
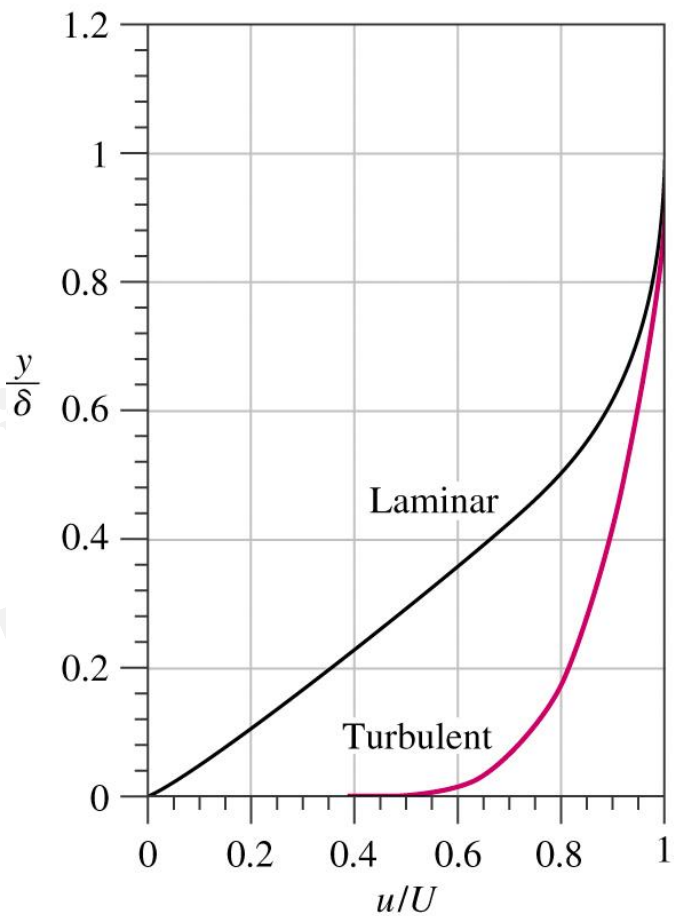


Illustration of unsteadiness of a turbulent BL



Comparison of laminar and turbulent BL profiles

Turbulent Boundary Layer

- All BL variables [$U(y)$, δ, δ^*, θ] are determined empirically.
- One common empirical approximation for the time-averaged velocity profile is the **one-seventh-power law**

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{1/7} \qquad y \leq \delta$$

$$\frac{U}{U_e} \cong 1 \qquad y > \delta$$

TABLE 10–4

Summary of expressions for laminar and turbulent boundary layers on a smooth flat plate aligned parallel to a uniform stream*

Property	Laminar	(a) Turbulent(†)	(b) Turbulent(‡)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

* Laminar values are exact and are listed to three significant digits, but turbulent values are listed to only two significant digits due to the large uncertainty affiliated with all turbulent flow fields.

† Obtained from one-seventh-power law.

‡ Obtained from one-seventh-power law combined with empirical data for turbulent flow through smooth pipes.

Results of Numerical Analysis

$$\delta \approx \frac{5.0}{\sqrt{U/\nu x}} = \frac{5.0x}{\sqrt{Re_x}}$$

$$\tau_w = \frac{0.332\rho U^2}{\sqrt{Re_x}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$$

Momentum Integral Equation

- Provides Approximate Alternative to Exact (Blassius) Solution

$$\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$$

Momentum Integral Equation

Equation is used to estimate the boundary-layer thickness as a function of *x*:

1. Obtain a first approximation to the free stream velocity distribution, $U(x)$. The pressure in the boundary layer is related to the free stream velocity, $U(x)$, using the Bernoulli equation

2. Assume a reasonable velocity-profile shape inside the boundary layer

3. Derive an expression for t_w using the results obtained from item 2

Pressure Gradients in Boundary-Layer Flow

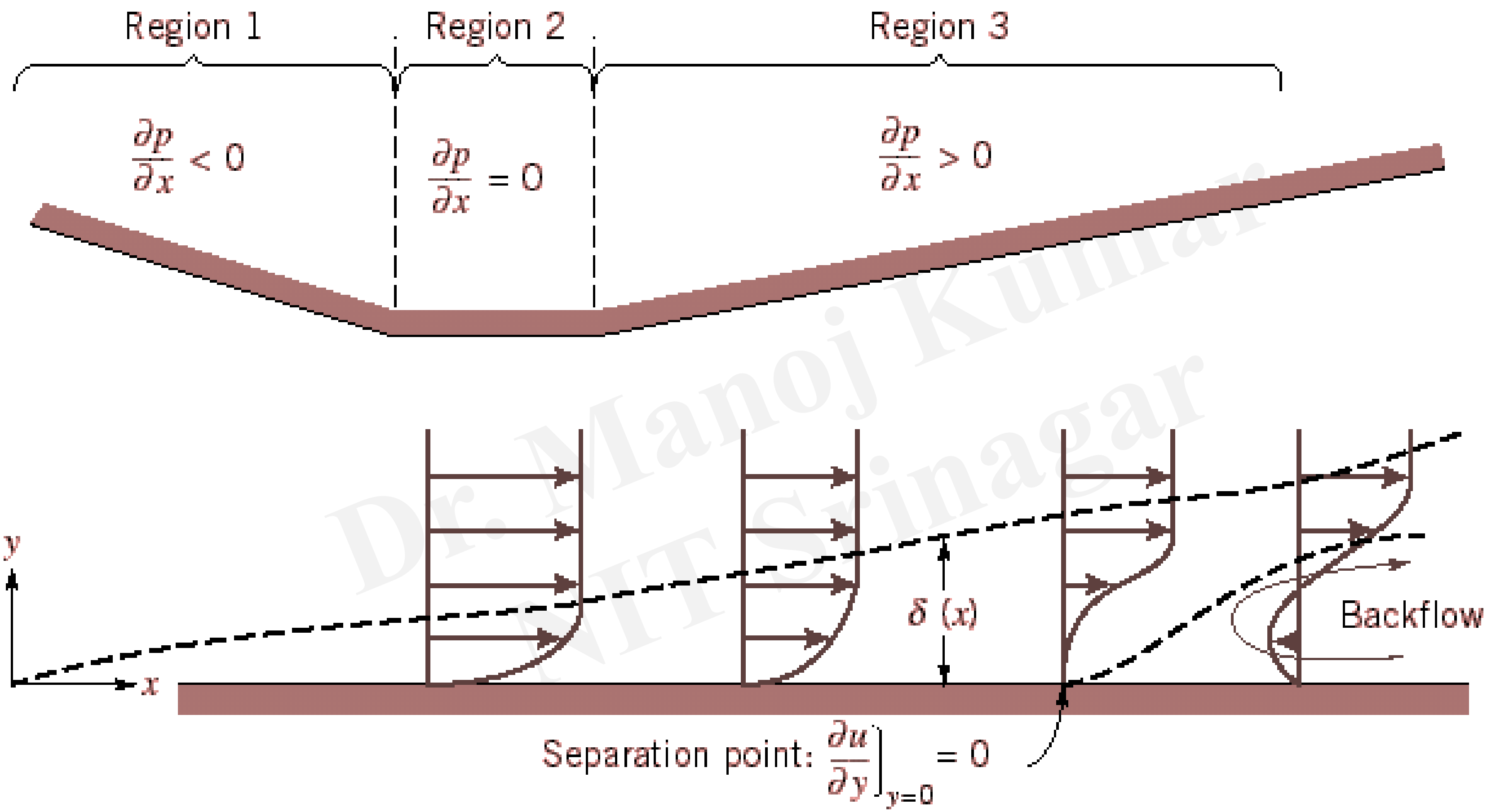
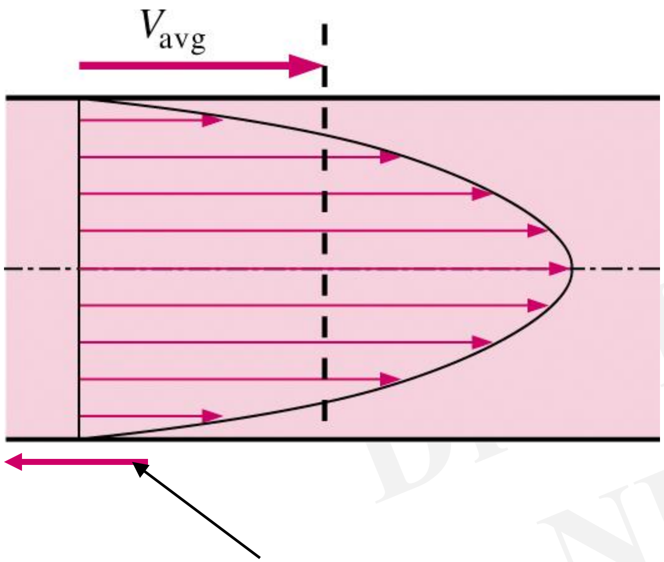


Fig. 9.6 Boundary-layer flow with pressure gradient (boundary-layer thickness exaggerated for clarity).

Introduction- Pipe Flow

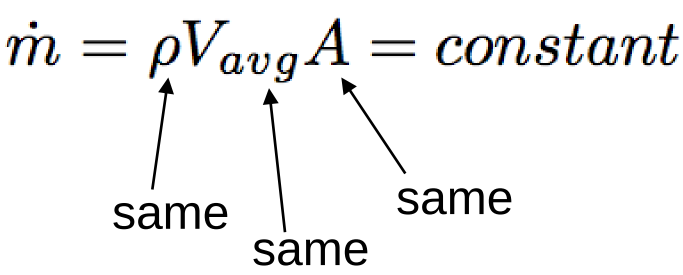


Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls

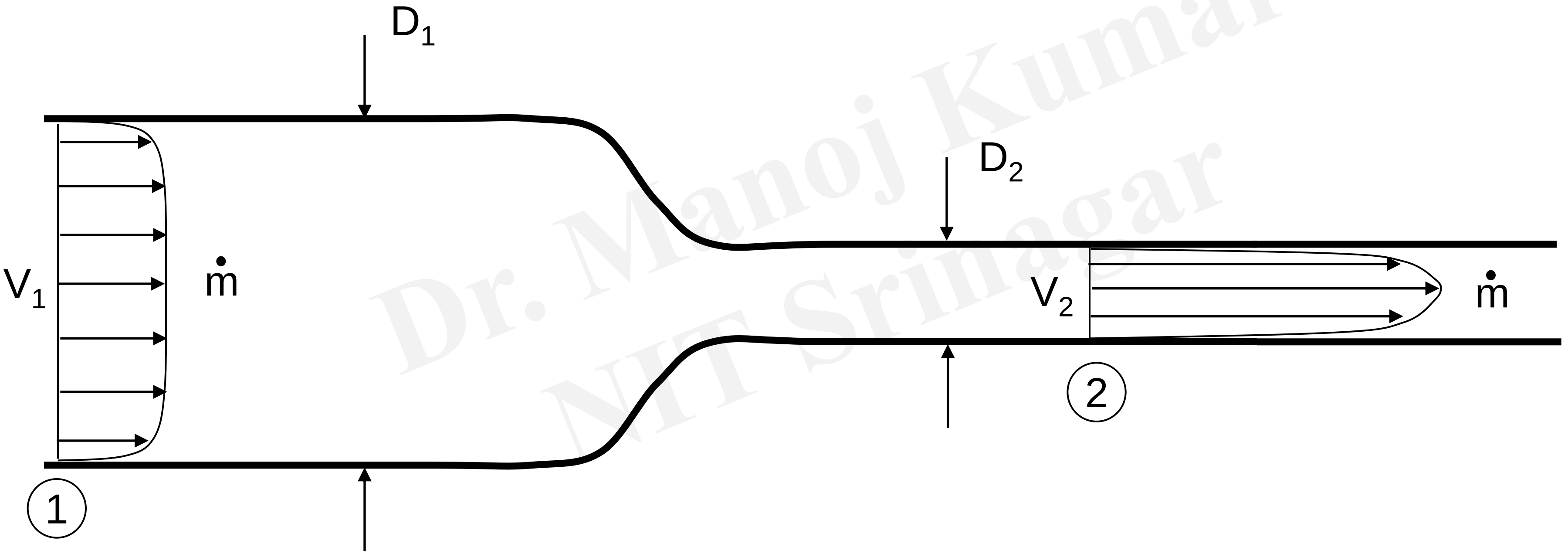
Introduction

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass



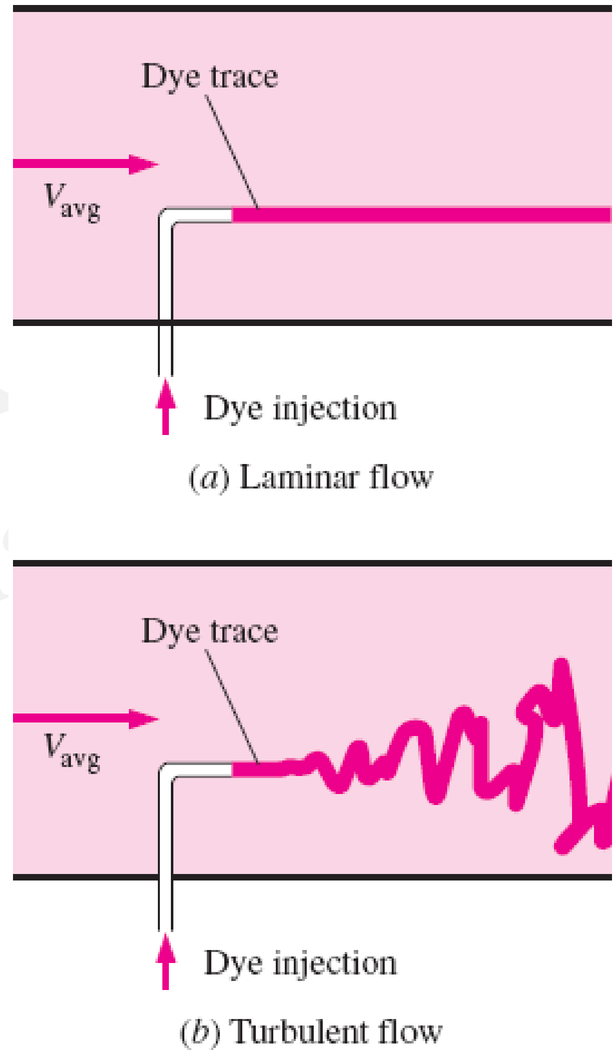
Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



LAMINAR AND TURBULENT FLOWS

- **Laminar flow:** characterized by *smooth streamlines* and *highly ordered motion.*
- **Turbulent flow:** characterized by *velocity fluctuations* and *highly disordered motion.*
- The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.



Reynolds Number

- The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*, among other things.
- British engineer Osborne Reynolds (1842–1912) discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid.
- The ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$

Reynolds Number

- At large Reynolds numbers, the inertial forces are large relative to the viscous forces ⇒ Turbulent Flow
- At *small* or *moderate* Reynolds numbers, the viscous forces are large enough to suppress these fluctuations ⇒ Laminar Flow
- The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number,** Re_{cr}.
- The value of the critical Reynolds number is different for different geometries and flow conditions. For example, $Re_{cr} = 2300$ for internal flow in a circular pipe.

Reynolds Number

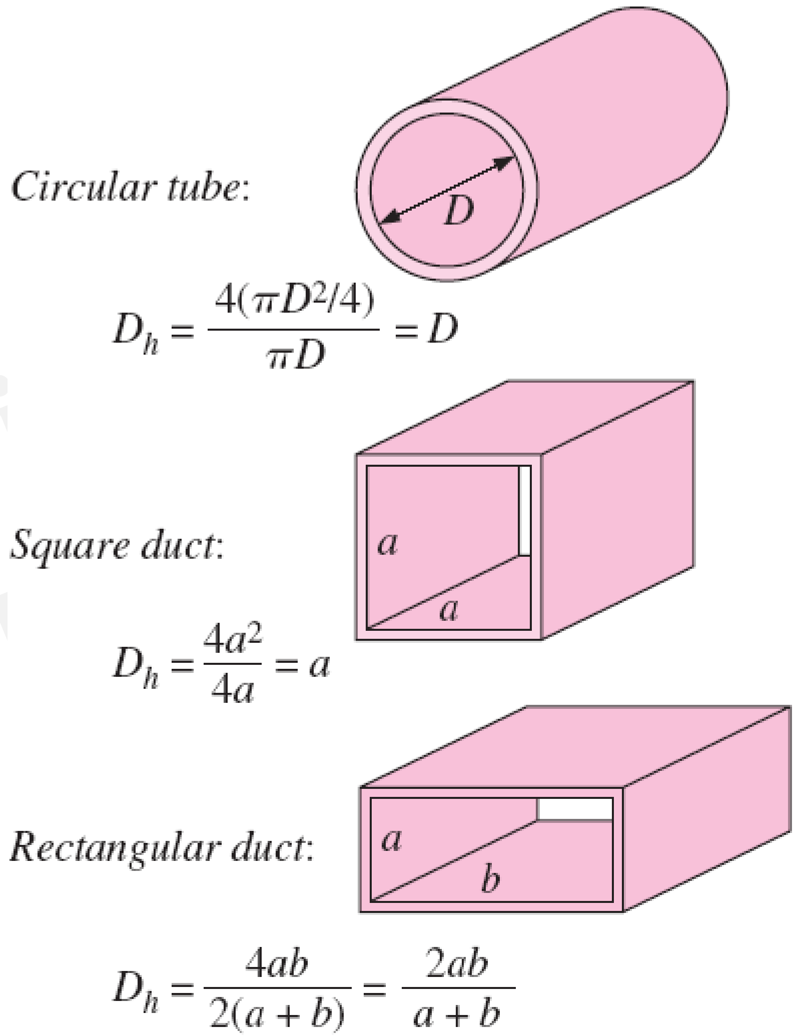
- For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as

$$D_h = \frac{4A_c}{p}$$

A_c = cross-section area

P = wetted perimeter

- The transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness*, *pipe vibrations*, and *fluctuations in the flow.*

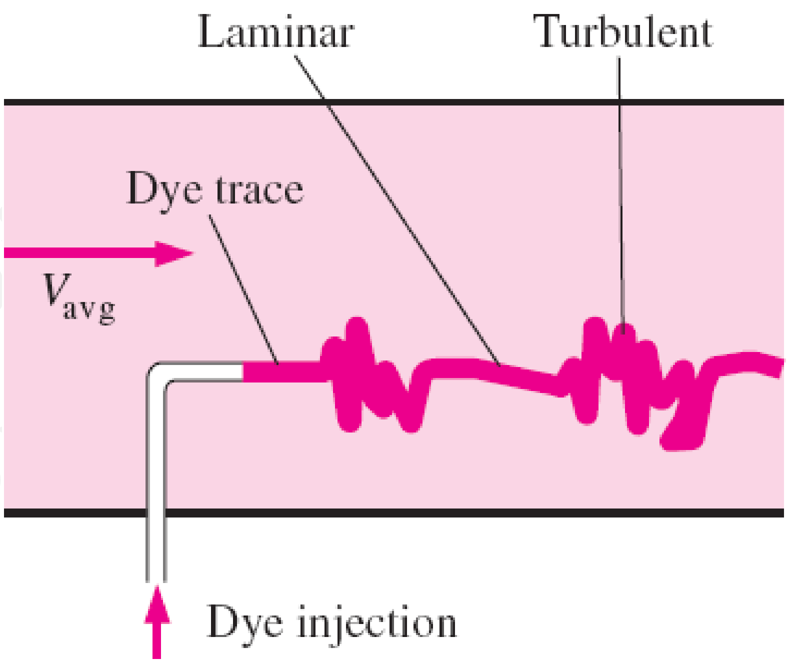


Reynolds Number

- Under most practical conditions, the flow in a circular pipe is

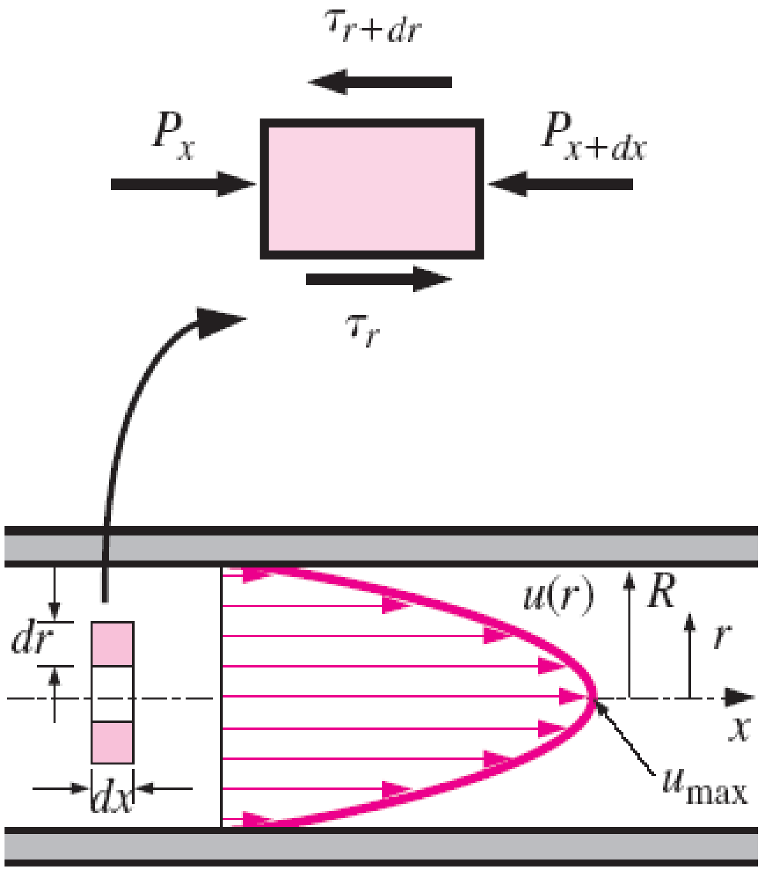
$Re \lesssim 2300$	laminar flow
$2300 \lesssim Re \lesssim 4000$	transitional flow
$Re \gtrsim 4000$	turbulent flow

- In transitional flow, the flow switches between laminar and turbulent randomly.



LAMINAR FLOW IN PIPES

- In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe.
- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and no motion in the radial direction such that no acceleration (since flow is steady and fully-developed).



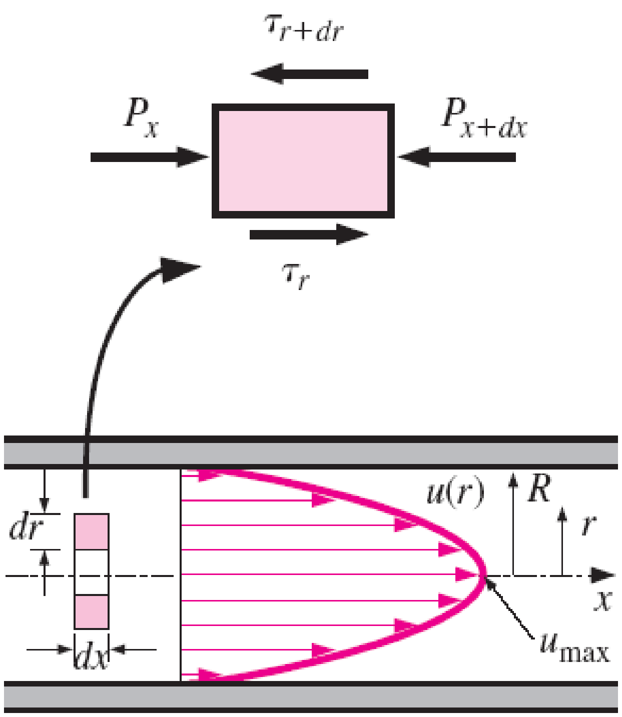
LAMINAR FLOW IN PIPES

- Now consider a ring-shaped differential volume element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with the pipe. A force balance on the volume element in the flow direction gives

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} +$$
$$(2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

- Dividing by 2p*drdx* and rearranging,

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$



LAMINAR FLOW IN PIPES

- Taking the limit as *dr*, *dx* → 0 gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

- Substituting t = -m(*du*/*dr*) gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

- The left side of the equation is a function of *r*, and the right side is a function of *x*. The equality must hold for any value of *r* and *x*; therefore, *f* (*r*) = *g*(*x*) = constant.

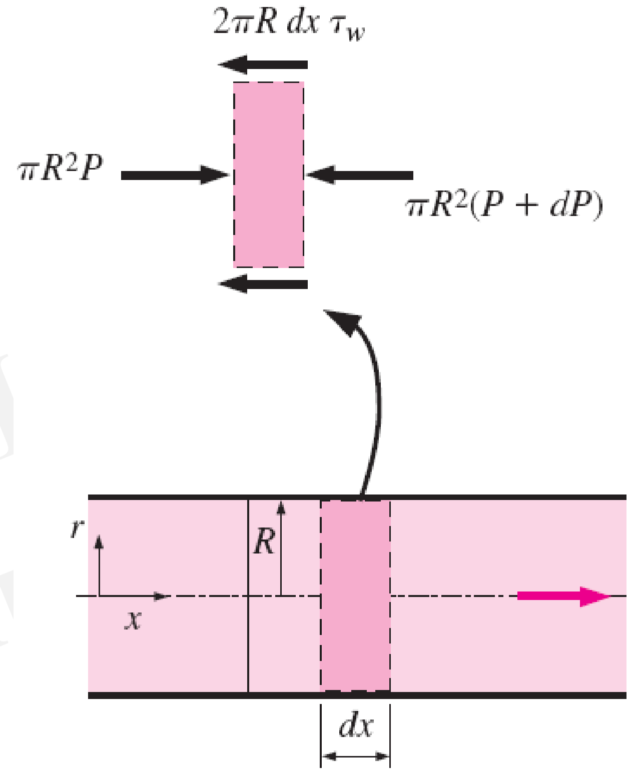
LAMINAR FLOW IN PIPES

- Thus we conclude that dP/dx = constant and we can verify that

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

- Here t_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Then we solve the u(r) eq. by rearranging and integrating it twice to give

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

LAMINAR FLOW IN PIPES

- *Since* $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$, *then we can get* u(r)

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

- Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic*. Since u is positive for any r, and thus the dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).
- The average velocity is determined from

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

LAMINAR FLOW IN PIPES

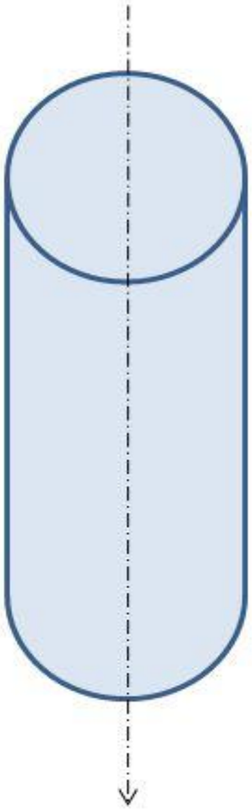
- The velocity profile is rewritten as

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

- Thus we can get

$$u_{max} = 2V_{avg}$$

- Therefore, *the average velocity in fully developed laminar pipe flow is one half of the maximum velocity*.



Poiseuille flow in a cylinder (Hagen-Poiseuille):

Assume: flow along O_z+ rotational invariance: $\underline{v}(r,\theta,z) = v_z(r,z)\underline{e}_z$

Continuity equation: $\dfrac{\partial v_z}{\partial z} = 0 \Rightarrow v_z(r)$

Boundary conditions: $\underline{v}(r=R,\theta) = \underline{0}$

$P(z=0) - P(z=L) \equiv \Delta P$ Pressure Gradient

Navier-Stokes equation:
$$\begin{cases} \dfrac{\partial P}{\partial r} = \dfrac{\partial P}{r\partial \theta} = 0 \Rightarrow P(z) \\ \eta\left(\dfrac{\partial^2 v_z}{\partial r^2} + \dfrac{1}{r^2}\dfrac{\partial^2 v_z}{\partial \theta^2} + \dfrac{1}{r}\dfrac{\partial v_z}{\partial r}\right) = \dfrac{\partial P}{\partial z} - \rho g \end{cases}$$

$$v_{z(r)} = \frac{\Delta P + \rho g L}{4\eta L}.(R^2 - r^2)$$

Flow rate: $Q = \dfrac{\Delta P + \rho g L}{8\eta L}.\pi.R^4$

Friction force: $F_z = (\Delta P + \rho g L)\pi.R^2$

Total pressure force: $F_r = \langle P \rangle 2\pi.R.L$

Pressure Drop and Head Loss

- The *pressure drop* ΔP of pipe flow is related to the power requirements of the fan or pump to maintain flow. Since dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

- The pressure drop for laminar flow can be expressed as

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

- ΔP due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss*.

Pressure Drop and Head Loss

- In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L.

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g}$$

(Frictional losses due to viscosity)

Friction Losses

The resulting pressure (energy and head) losses are usually computed through the use of modified Fanning's friction factors:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}}$$

where F_k is the characteristic force, S is the friction surface area. This equation is general and it can be used for all flow processes.

Used for a pipe:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)\frac{D^2\pi}{4}}{(D\pi L)\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)D}{2L\rho v^2} = \frac{\Delta p}{L}\frac{D}{2\rho v^2}$$

where Fk is the press force, S is the area of curved surface. Rearranged, we get a form of pressure loss:

$$\Delta p_L = 4f\frac{L}{D}\frac{v^2\rho}{2} = \lambda\frac{L}{D}\frac{v^2\rho}{2} = \zeta\frac{v^2\rho}{2}$$

Determination of Friction Factor with Dimensional Analysis

The Funning's friction factor is a function of Reynolds number, f = f(Re):

$$Re = \frac{vD}{\nu} = \frac{vD\rho}{\mu}$$

Many important chemical engineering problems cannot be solved completely by theoretical methods. For example, the pressure loss from friction losses in a long, round, straight, smooth pipe depends on all these variables: the length and diameter of pipe, the flow rate of the liquid, and the density and viscosity of the liquid.

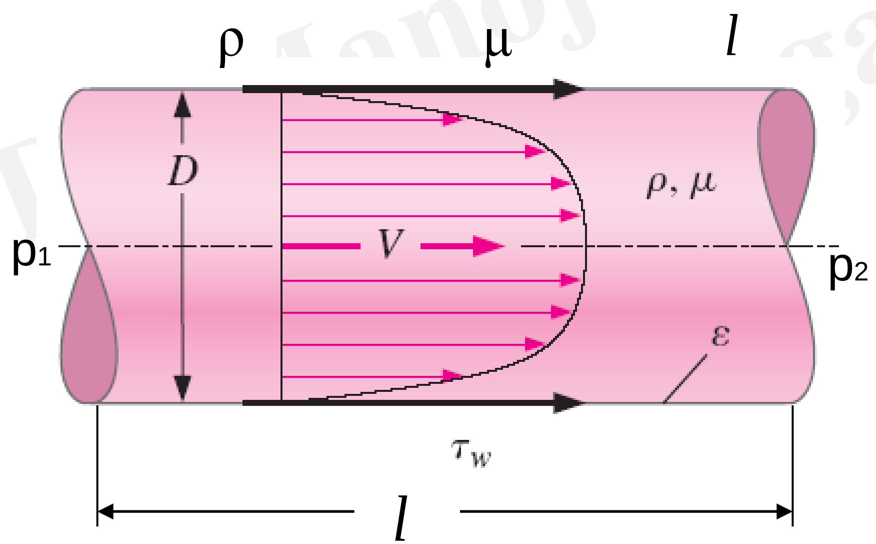
If any one of these variables is changed, the pressure drop also changes. The empirical method of obtaining an equation relating these factors to pressure drop requires that the effect of each separate variable be determine in turn by systematically varying that variable while keeping all others constant.

It is possible to group many factors into a smaller number of dimensionless groups of variables. The groups themselves rather than separate factors appear in the final equation. These method is called dimensional analysis, which is an algebric treatment of the symbols for units considered independtly of magnitude.

Determination of Friction Factor with Dimensional Analysis

Many important chemical engineering problems cannot be solved completely by theoretical methods. For example, the pressure loss from friction losses (or the pressure difference between two ends of a pipe) in a long, round, straight, smooth pipe a fluid is flowing depends on all these variables: pipe diameter d, pipe length , fluid velocity v, fluid density , and fluid viscosity .

$$\Delta p = p_1 - p_2$$



The relationship may be written as: $$\Delta p = f\left(D, l, v, \rho, \mu\right) \qquad (1)$$

The form of the function is unknown, but since any function can be expanded as a power series, the function can be regarded as the sum of a number of terms each consisting of products of powers of the variables. The simplest form of relations will be where the function consists simply of a single term, when:

The requirement of dimensional consistency is that the combined term on the right-hand side will have the same dimensions as that the on the left, i.e. it must have the dimensions of pressure.

Each of the variables in equation (2) can be expressed in terms of mass, length, and time. Thus, dimensionally:

$$\Delta p = \text{const}\, D^a l^b v^c \rho^d \mu^e \qquad (2)$$

$$\Delta p = ML^{-1}T^{-2} \qquad v = LT^{-1}$$

$$D = L \qquad \rho = ML^{-3}$$

$$l = L \qquad \mu = ML^{-}T^{-1}$$

i.e.:

$$ML^{-1}T^{-2} = L^a L^b (LT^{-1})^c (ML^{-3})^d (ML^{-1}T^{-1})^e$$

The conditions of dimensional consistency must be met for the fundamentals of M, L, and T and the indices of each of these variables can be equated. Thus:

In

$$\begin{aligned} &M \quad 1 = d + e \\ &L \quad -1 = a + b + c - 3d - e \\ &T \quad -2 = -c - e \end{aligned}$$

Thus three equations and five unknowns result and the equations may be solved in terms of any two unknowns. Solving in terms of b and e:

$$d = 1 - e \quad (\text{from the equation in } M)$$

$$c = 2 - e \quad (\text{from the equation in } T)$$

Substituting in the L equation:

$$-1 = a + b + (2 - e) - 3(1 - e) - e$$

$$0 = a + b + e$$

$$a = -b - e$$

Thus, substituting into equation (2):

$$\Delta p = \text{const}\, D^{-b-e} l^{b} v^{2-e} \rho^{1-e} \mu^{e} =$$

$$= \text{const}\, D^{-b} D^{-e} l^{b} v^{2} v^{-e} \rho \rho^{-e} \mu^{e} =$$

$$= \text{const}\left(D^{-1} l\right)^{b} \left(D v \rho \mu^{-1}\right)^{-e} \left(v^{2} \rho\right)$$

i.e.

$$\frac{\Delta p}{\rho v^{2}} = \text{const}\left(\frac{l}{D}\right)^{b} \left(\frac{D v \rho}{\mu}\right)^{-e}$$

Let: $\text{const} = \dfrac{k}{2}$

Thus: $$\frac{\Delta p}{\rho v^{2}} = \frac{k}{2}\left(\frac{l}{D}\right)^{b} \mathrm{Re}^{-e} \longrightarrow \Delta p = \frac{k}{\mathrm{Re}^{e}}\left(\frac{l}{D}\right)^{b} \frac{\rho v^{2}}{2}$$

b=1, and k and e have to determinate by experiments.

For laminar flow k=64 and e=1

For turbulent flow k=0,0791 and e=0,25.

$$\Delta p = \frac{k}{\mathrm{Re}^{e}} \frac{l}{D} \frac{\rho v^{2}}{2} = 4f \frac{l}{D} \frac{\rho v^{2}}{2}$$

If a theoretical equation for this problem exist, it can be written in the general form. List of relevant parameters:

$$\frac{\Delta p}{L} = f\left(D, v, \rho, \mu\right)$$

If Eq.1. is a valid relationship, all terms in the function f must have the same dimensions as those of the left-hand side of the equation $\Delta p/L$.

Let the phrase the dimensions of be shown by the use of brackets. Then any term in the function must conform to the dimensional formula

$$\frac{\Delta p}{L} = \text{const.} D^{a} v^{b} \rho^{c} \mu^{d}$$

$$\frac{N}{m^{2} \cdot m} = (m)^{a} \left(\frac{m}{s}\right)^{b} \left(\frac{kg}{m^{3}}\right)^{c} \left(\frac{kg}{ms}\right)^{d}$$

$$MT^{-2}L^{-2} = L^{a}\left(LT^{-1}\right)^{b}\left(ML^{-3}\right)^{c}\left(ML^{-1}T^{-1}\right)^{d}$$

$$MT^{-2}L^{-2} = L^{a}\left(L^{b}T^{-b}\right)\left(M^{c}L^{-3c}\right)\left(M^{d}L^{-d}T^{-1}\right)$$

M: $1 = c+d$

L: $-2 = a+b-3c-d$

T: $-2 = -b-d$

M: c=1-d

T: b=2-d

L: a=-2-b+3c+d=-2-2+d+3-3d+d

a=-1-d

$$\frac{\Delta p}{L} = const \cdot D^{-1-d} v^{2-d} \rho^{1-d} \eta^{d}$$

$$f = \frac{A}{Re^{d}}$$

$$\frac{\Delta p}{L} = const. \cdot \left(\frac{Dv\rho}{\eta}\right)^{-d} \frac{v^2\rho}{D}$$

$$\Delta p = f \cdot \frac{L}{D} \cdot \frac{v^2\rho}{2}$$

$$\frac{\Delta p}{L} = A \cdot \left(\frac{Dv\rho}{\eta}\right)^{-d} \cdot \frac{1}{D} \cdot \frac{v^2\rho}{2}$$

Fluid Flow in Pipes

Goals: determination of friction losses of fluids in pipes or ducts, and of pumping power requirement.

The resulting pressure (energy and head) loss

$$\Delta p_L = (z_1 - z_2)\rho g + (p_1 - p_2) + \frac{(v_1^2 - v_2^2)\rho}{2}$$

is usually computed through the use of the modified Fanning friction factor:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}}$$

Used for a pipe:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)\frac{D^2\pi}{4}}{(D\pi L)\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)D}{2L\rho v^2} = \frac{\Delta p}{L}\frac{D}{2\rho v^2}$$

where F_k is the press force, S is the area of curved surface. Rearranged, we get a form of pressure loss:

$$\Delta p_L = 4f\frac{L}{D}\frac{v^2\rho}{2} = \lambda\frac{L}{D}\frac{v^2\rho}{2} = \zeta\frac{v^2\rho}{2}$$

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